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## *Department of PHYSICS*

## II Year B.Sc.-Physics: IV Semester

# Paper 4: ELECTRICITY MAGNETISM ELECTRONICS

## STUDY MATERIAL

**Name of the Student :** ________________________________

**Roll Number :** ________________________________

**Group :** ________________________________

**Academic Year :** ________________________________

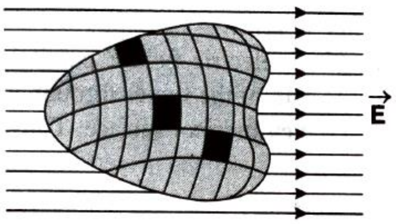
# UNIT-I

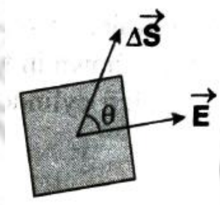
## 1.1 ELECTRIC FIELD and ELECTRIC POTENTIAL

### ELECTRIC FLUX

*The electric flux through a surface placed inside electric field represents the total number of electric lines of force crossing the surface in a direction normal to the surface.*

Let the surface be divided into a number of elementary squares. Each square on the surface may be represented by a vector d***S*** whose magnitude is equal to its area and the direction taken as the *outward normal* drawn on this surface. Let **E** be the electric field vector acting on the surface.




*The scalar product, i.e.,* E. d*S is defined as the electric flux for the surface.* The total flux $\Phi_E$ through the entire surface is given by

$$\Phi_E = \oint \boldsymbol{E}.d\boldsymbol{S} = E.S$$

If θ is the angle between E and d*S*, then the scalar product is given by

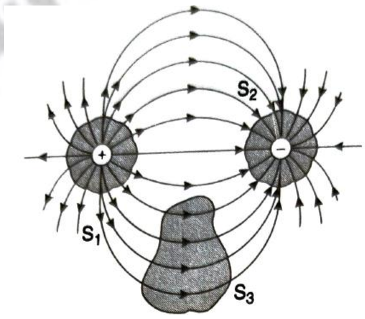
**E**. ***dS*** = E*dS* cos θ

Now the electric flux $\Phi_E = \oint EdS \cos\theta$

$$\Phi_E = E\cos\theta \oint dS$$

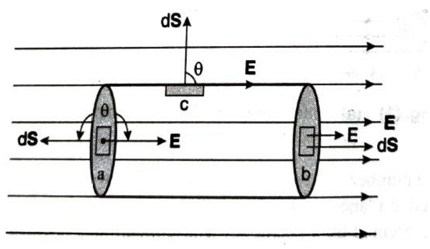
$$\Phi_E = EA\cos\theta$$

For any closed surface, flux is taken as positive if they are outwards and taken as negative if inwards.




**Example 1:** From the figure, for the surfaces $S_1$and $S_2$ the electric flux are outwards and inwards so conveniently it is taken as positive for $S_1$ and negative for $S_2$. But for surface $S_3$, it is zero because the amount of flux entering is equal to leaving the closed surface area $S_3$.

**Example 2:** Consider a cylinder of radius *R* immersed in a uniform electric field E *parallel* to its surface. The flux $\Phi_E$ for the entire cylinder is the sum of the fluxes through (a) left face (left cylinder cap) (b) right face (right cylinder cap) and (c) the cylindrical surface.




Thus,

For the left cap of the cylinder, the angle between E and *dS* is 180°

$$\int E.dS = \int EdS\cos 180° = -E\int dS = -ES$$

Similarly, for the right cap of the cylinder, the angle between E and $dS$ is zero

$$\int E.dS = \int EdS\cos\,0° = E\int dS = ES$$

For the curved surface of the cylinder, the angle between E and $dS$ is 90°

$$\int E.dS = \int EdS\cos\,90° = \int EdS(0) = 0$$

So, the total flux through the entire cylinder

$$\Phi_E = -ES + ES + 0 = 0$$

**GAUSS'S LAW**

**Statement:**

*Gauss law states that total normal electric flux $\phi_E$ over a closed surface in an electric field is* $(1/\varepsilon_0)$ *times the total charge Q enclosed within the surface.*

Mathematically, it can be expressed as

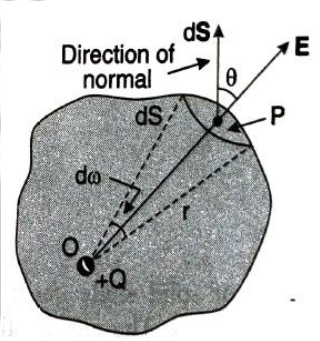
$$\Phi_E = \oiint E.dS = \oiint EdS\cos\theta = \left(\frac{1}{\varepsilon_0}\right)Q$$

where $\varepsilon_0$ is the permittivity of the free space.

**Proof:**

(i) When the charge is within the surface

Let a charge $+Q$ be placed at $O$ within a closed surface of irregular shape. Consider a point P on the surface at a distance $r$ from $O$. Now take a small area $dS$ around $P$. The normal to the surface $dS$ is represented by a vector $dS$ which makes an angle θ with the direction of electric field E along $OP$. The electric flux $d\Phi_E$ outwards through the area $dS$ is given by




$$d\Phi_E = E \cdot dS = E\,dS\cos\theta \qquad (1)$$

where θ is angle between E and $dS$.

From Coulomb's law, the electric intensity $E$ *at* a point $P$ *of distance* $r$ from a point charge $Q$ is given by

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r^2}\right) \qquad (2)$$

From eqs. (1) and (2), we get

$$d\Phi_E = \frac{Q}{4\pi\varepsilon_0 r^2}\,.dS\cos\theta$$

$$d\Phi_E = \frac{Q}{4\pi\varepsilon_0}\left(\frac{dS\cos\theta}{r^2}\right)$$

But $\left(\frac{dS\cos\theta}{r^2}\right)$ is the solid angle $d\omega$ subtended by $dS$ at $O$.

Hence, $$d\Phi_E = \frac{Q}{4\pi\varepsilon_0}d\omega \qquad (3)$$

The total flux $\Phi_E$ over the entire whole surface is given by

$$\Phi_E = \frac{Q}{4\pi\varepsilon_0}\oint d\omega$$

where $\oint d\omega$ is the solid angle subtended by the whole surface at $O$. This is equal to 4 $\pi$. Hence,

$$\Phi_E = \frac{Q}{4\pi\varepsilon_0} \text{ x } 4\pi$$

$$\therefore \quad \Phi_E = \frac{Q}{\varepsilon_0} \qquad (4)$$

Let the closed surface enclose several charges $+Q_1$, $+Q_2$, $+Q_3$, +,…, $-Q_1' - Q_2'$, $-Q_3'$, …

Now each charge will contribute to the total electric flux. Thus, the total flux is given by

$$\Phi_E = \frac{1}{\varepsilon_0}\left[+Q_1 + Q_2 + Q_3 + \ldots - Q_1' - Q_2' - Q_3' \ldots\right]$$

$$\Phi_E = \frac{1}{\varepsilon_0}\sum Q \qquad (5)$$

where $Q$ is algebraic sum of all the charges.

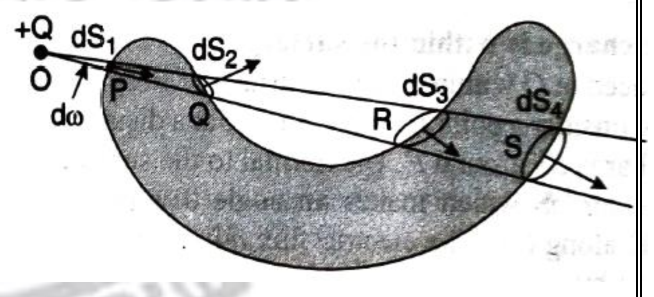
So the total normal electric flux over the closed surface is equal to $(1/\varepsilon_0)$ times the total charge enclosed within the surface. Hence Gauss law is proved.

<u>(ii) When the charge is outside the surface.</u>

Let a point charge $+Q$ be situated at point $O$ outside the closed surface. Now cone of solid angle $d\omega$ from $O$ cuts the surface areas $dS_1$, $dS_2$, $dS_3$, $dS_4$ at points P, $Q$, R and S respectively. The electric flux for an outward normal is positive while the inward drawn normal is negative. Therefore, the flux through areas, $dS_2$ and $dS_4$ are positive while for $dS_1$ and $dS_3$ are negative. Therefore,

the electric flux at $P$ through area $dS_1 = \left(\frac{-Q}{4\pi\varepsilon_0}\right)d\omega$

the electric flux at $Q$ through area $dS_2 = \left(\frac{+Q}{4\pi\varepsilon_0}\right)d\omega$




the electric flux at $R$ through area $dS_3 = \left(\frac{-Q}{4\pi\varepsilon_0}\right)d\omega$

the electric flux at $S$ through area $dS_4 = \left(\frac{+Q}{4\pi\varepsilon_0}\right)d\omega$

$\therefore$ Total electric flux $= \left(\frac{-Q}{4\pi\varepsilon_0}\right)d\omega + \left(\frac{+Q}{4\pi\varepsilon_0}\right)d\omega + \left(\frac{-Q}{4\pi\varepsilon_0}\right)d\omega + \left(\frac{+Q}{4\pi\varepsilon_0}\right)d\omega = 0$

So the total electric flux over the whole surface due to an external charge *is* **zero.** This verifies Gauss's law.

## <u>ELECTRIC FIELD</u>

*The region surrounding an electric charge on a group of charges, in which another charge experiences a force is called electric field.*

The force between two charges can be considered in terms of electrical fields as follows:

(i) A charge $q_1$ sets up an electric field in space itself.

(ii) The field of $q_1$ acts on $q_2$, now, $q_2$ experiences a force due to $q_1$.

(iii) Similarly $q_2$ sets up the field and this fields acts on $q_1$ thus, produces a force on $q_1$

**INTENSITY OF ELECTRIC FIELD E.**

The intensity of electric field at a point in the field is defined as the force experienced by a unit positive charge placed at that point.

Let F be the force experienced by a test charge $q_0$ placed at a point in the electric field, then the intensity of electric field E at that point is given by

$$\mathbf{E} = \frac{\mathbf{F}}{q_0} \frac{\text{newton}}{\text{coulomb}}$$

The intensity of electric field is vector quantity. The unit of electric field is newton per coulomb.

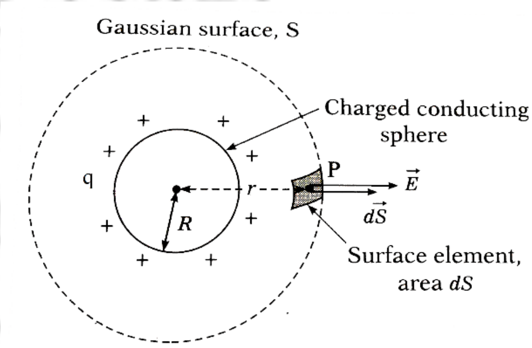
**APPLICATIONS OF GAUSS LAW**

**(1) Electric field intensity due to uniformly charged sphere**

Consider a sphere of radius R with center O. Let the charge q be uniformly distributed over it. Now the electric field at point P can be calculated as

*Case I: If the point P lies outside the sphere:*

Consider a point P outside the sphere at a distance r from the center O of the sphere. Now we have to find the field intensity E at this point. For this purpose a Gaussian sphere of radius r is constructed. The intensity of the field at all points on the Gaussian surface is constant and direction will be perpendicular to the surface.




Consider small area d**S** around P. The electric flux through with small area is given by

$$d\Phi_E = \boldsymbol{E} \cdot d\boldsymbol{S} = E\, dS \cos\theta$$

This is equal to the flux through the small area 'dS'. But in this case θ = 0, cos 0 = 1.

The total electric flux over a Gaussian surface is given by

$$d\Phi_E = EdS$$

The total normal electric flux over the entire Gaussian Surface is given by

$$\Phi_E = \int d\Phi_E = \int EdS$$

$$\Phi_E = E\int dS$$

$$\Phi_E = E\left(4\pi r^2\right) \qquad (1)$$

According to Gauss theorem

$$\Phi_E = \frac{q}{\varepsilon_0} \qquad (2)$$

From (1) and (2)

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

*<u>Case II : If the point P lies on the surface of the sphere:</u>*

In this case r = R, Now the electric field at P on the surface,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

*<u>Case III: If the point P lies inside the sphere:</u>*

Consider a point ‘P’ inside the sphere at a distance r from the center ‘O’ of the sphere. Now we have to find the field intensity ‘E’ at this point. For this purpose we construct a Gaussian sphere of radius ‘r’. Consider a small area ‘dS ‘around ‘P’. The electric flux through with small area is given by

$$d\Phi_E = E.dS = EdS \qquad (\because \cos 0 = 1)$$

The total electric flux over a Gaussian surface is given by

$$\Phi_E = \int d\Phi_E = \int EdS$$

$$\Phi_E = E\int dS$$

$$\Phi_E = E\left(4\pi r^2\right) \qquad (3)$$

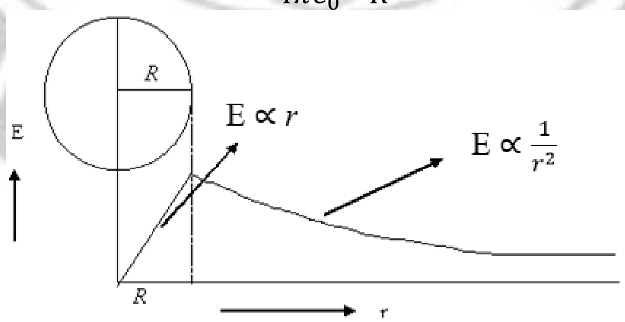
According to Gauss theorem

$$\Phi_E = \frac{q}{\varepsilon_0} = \frac{\left(\frac{4}{3}\pi r^3 \rho\right)}{\varepsilon_0} = \frac{\left(\frac{4}{3}\pi r^3 \times \frac{q}{\frac{4}{3}\pi R^3}\right)}{\varepsilon_0} = \frac{q}{\varepsilon_0}\left(\frac{r^3}{R^3}\right) \qquad (4)$$

From (3) and (4)

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}\left(\frac{r^3}{R^3}\right)$$

$$E = \frac{q}{4\pi\varepsilon_0}\left(\frac{r}{R^3}\right)$$



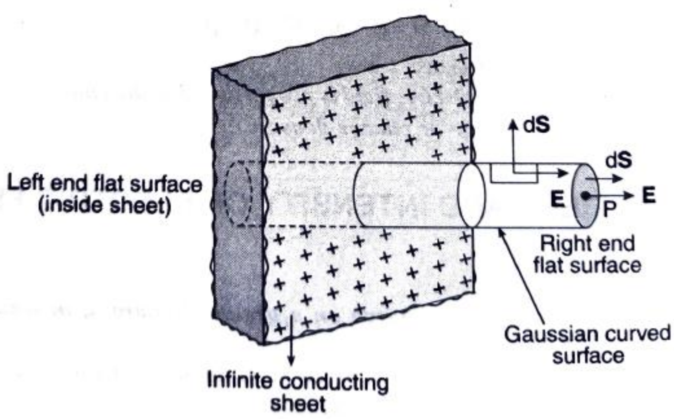

The variation of ‘**E**’ with distance ‘**r**’ as shown in figure.

**<u>(2) Field intensity due to a conducting infinite thin sheet of charge:</u>**

Consider an infinite charged conducting sheet. As the sheet is conducting, the charge will reside only on the surface of the conductor. There will be no charge inside the sheet. Let σ be the charge density (charge per unit area). We have to determine the electric field at a point P, near the surface and outside of sheet. For this we construct a cylindrical Gaussian surface passing through a point P.

The Gaussian surface has one flat plane surface passing through P at right end and other flat surface just inside the surface of conductor. There is a curved surface being perpendicular to the surface of conductor.

From figure electric flux Φ through the entire Gaussian cylinder is the contribution of three surfaces.




At the right end, E is parallel to dS, i.e., $\Phi_{E_{\text{right}}} = EdS$

At the left end there is no electric field inside conductor. i.e., $\Phi_{E_{\text{left}}} = 0$.

For the curved surface E is perpendicular to dS, i.e., $\Phi_{E_{\text{curved}}} = 0$.

So, we have

$$\Phi = \underset{\substack{\text{right}\\\text{end}}}{\oint} \mathbf{E} \bullet d\mathbf{S} + \underset{\substack{\text{left}\\\text{end}}}{\oint} \mathbf{E} \bullet d\mathbf{S} + \underset{\substack{\text{curved}\\\text{surface}}}{\oint} \mathbf{E} \bullet d\mathbf{S}$$

$$\Phi = \oint E\, dS \cos 0^\circ + \oint 0 \cdot dS \cos 0^\circ + \oint E\, dS \cos 90^\circ$$

$$\Phi = E\, S + 0 + 0 = E\, S$$

According to Gauss law,

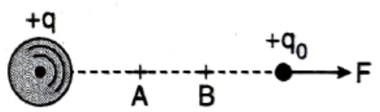
$$E\, S = \frac{q}{\varepsilon_0} = \frac{\sigma\, S}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

***Thus,** the electric field intensity outside the infinite conducting sheet of charge is equal to* $(1/\varepsilon_0)$ times the surface charge density.

## ELECTRIC POTENTIAL

The electric field around a group of charges can be described by electric field strength **E**. This can also be described by a scalar quantity known as electric potential. Electric intensity and electric potential are intimately related to each other.




Consider an electric field due to a positive charge +q as shown in figure. Let there be a positive test charge $+q_0$ in this field. This test charge experiences an electric repulsive force F due to +q. Suppose the test charge is moved from a point B to another point A. Now some work has to be done against the force of repulsion. The ratio of work done in taking a test-charge from one point to other point in an electric field to the magnitude of the test charge is defined as the electric potential difference between these points.

If W be the work done in moving the test charge $q_0$ from point B to point A, then the potential difference between A and B is

$$V_A - V_B = \frac{W}{q_0}.$$

Suppose the point B (taken as reference point) is chosen at infinity, then the electric potential $q_0$ at this infinite is arbitrarily taken as zero.

$$V_A = \frac{W}{q_0},$$

where W is the work done by an external agent to carry the test charge $q_0$ from infinity to point A.

Thus, *electric potential at a point in the electric field is defined as the work done by an external agent in carrying a unit positive test charge from infinity to that point against the electric force of the field.*

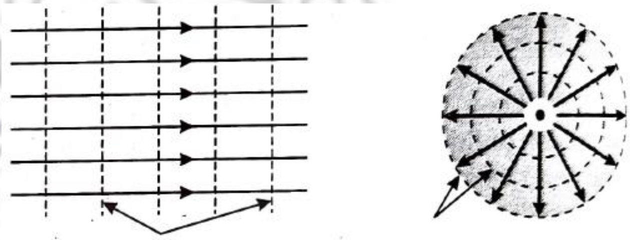
SI units: volt

One volt is defined as the difference in potential between the points so that one joule of work is done in carrying one coulomb of positive charge from one point to other (1 volt = 1 joule/1 coulomb).

**<u>EQUIPOTENTIAL SURFACES</u>**

Equipotential surface in an electric field is a surface on which the potential is same at every point. In other words, the locus of all points which have the same electric potential is called *equipotential surface*. As the potential difference between any two points on the equipotential is zero, hence no work is done in taking a charge from one point to another. This is only possible when the charge is taken perpendicular to the field. In this way, the equipotential surface at every point is perpendicular to the field (lines of force).

Eg: (a) In case of uniform field, where the lines of forces are straight and parallel, the equipotential surfaces are planes perpendicular to the lines of forces.



(a) Equipotential Surfaces (b)

Eg: (b) For a sphere of charge or a point charge, the equipotential surfaces are a family of concentric spheres.

Equipotential surfaces in electrostatics are similar to wavefronts in optics. The wave fronts in optics are the locus of all points which are in the (a) Equipotential Surfaces (b) Same phase. These planes perpendicular to the direction of rays. On the other hand, the equipotential surfaces are perpendicular to the lines of force.

In case of equipotential surfaces

(i) The electric field along the equipotential surface is zero.

(ii) The electric field is always normal to the surface.

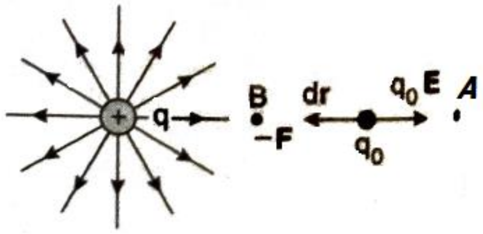
(iii) The work done in moving a charge on the equipotential surface is zero.

Work done = potential difference x charge = 0 x charge = zero.

(iv) When the charge is infinite, the equipotential surface is plane.

(v) The equipotential surfaces act as wave-fronts in optics.

## POTENTIAL DUE TO A POINT CHARGE




Consider a point charge +q whose electric field E is outward along radial line. Consider two points A and B along radial line (for convenience). Let a test charge $q_0$ be moved from reference point A to B.

The force exerted by the field of charge q on test charge go is $q_0 E$.

Now to move the test charge go towards B, a force - $q_0 E$ must be applied. The work done by external agent to move the charge $\mathbf{q_0}$ through a small distance dr is given by

$$dW = q_0 [\mathbf{E} \cdot d\mathbf{r}] = q_0 E \, dr \cos 180°$$

$$dW = -q_0 E \, dr$$

$$dW = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{q q_0}{r^2} \cdot dr \qquad \left[\because \; E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\right]$$

Now the total work done in moving the test charge $\mathbf{q_0}$ from A to B is

$$W_{AB} = \int_{r_A}^{r_B} -\frac{1}{4\pi\varepsilon_0} \cdot \frac{q q_0}{r^2} \cdot dr$$

Where $r_A$ is the distance from A to B

$$= -\frac{q q_0}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r_A}^{r_B} = \frac{q q_0}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right].$$

The Potential difference between two points will be

$$V_B - V_A = \frac{W_{AB}}{q_0} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

To find the potential at point B, the reference point A is taken at infinity so that $V_A = 0$. Hence,

$$V_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r_B}$$

On dropping the suffix, the required expression becomes

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}.$$

This expression shows that at a distance r on all sides of charge q, the potential is the same. So for an isolated charge equipotential surfaces are sphere concentric with the point charge.

## ELECTRIC POTENTIAL DUE TO CHARGED CONDUCTING SPHERICAL SHELL

A spherical shell is a conducting hollow sphere of negligible thickness. Let R be the radius of spherical shell and σ be the charge density (charge per unit area). The total charge (q) on the spherical shell is given by

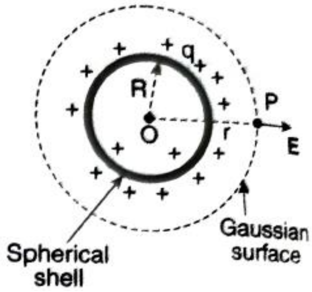
$$q = \text{area x charge density} = (4\,\pi R)^2\,\sigma$$

Case 1: When a point lies outside the shell

Let P be an external point at a distance r from the centre O of spherical shell. To calculate the electric potential at point P, draw a spherical shell (Gaussian surface) with centre O and radius OP = r as shown by dotted line. The electric field strength E at every point on the spherical shell will have the same magnitude and direction along the outward normal drawn to the surface at that point. The total normal electric flux over the whole Gaussian surface will be

$$\Phi = \oint \mathbf{E} \cdot d\mathbf{S} = \oint E\, dS \cos 0° = 4\pi r^2 \cdot E$$

According to Gauss's law,




$$E(4\pi r^2) = \frac{q}{\varepsilon_0} \quad \text{or} \quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

We know that $E = -\dfrac{\partial V}{\partial r}$ or $V = -\int E \cdot dr$

Substituting the value of E above eq., we get

$$V = -\int \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\, dr \quad \text{or} \quad V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

Case 2: When the point lies on the surface of the shell

In this case r = R (radius of the shell)

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}$$

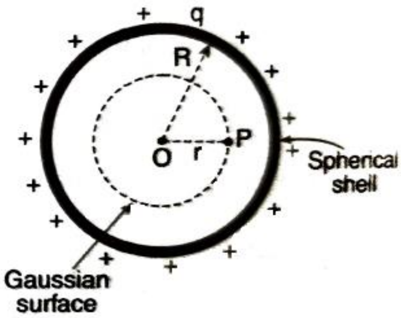
Now, the electric potential on the surface of the shall is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$$

Case 3: When the point lies inside the shell

Let P be a point inside the hollow spherical shell as shown in shell. Now draw Gaussian spherical surface with centre O and radius = OP. The Gaussian surface is shown by dotted line.

The charge enclosed by Gaussian surface is zero. Therefore, the electric field inside hollow conducting spherical shell is zero.

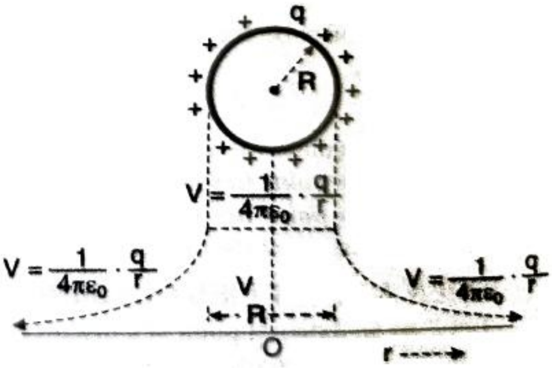



$$\because \quad E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0} = \frac{0}{\varepsilon_0} \quad \text{or} \quad E = 0$$

Therefore, the potential inside the shell is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$$

Inside the shell, the potential is constant and this is same as on the surface of the shell. The variation of electric potential due to a charged conducting spherical shell is shown in figure.




**POTENTIAL DUE TO UNIFORMLY CHARGED SPHERE (Non-Conducting Sphere)**

We know that the expressions of electric field in case of uniformly charged sphere are

Case 1: At a point outside the charged sphere, the electric field

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2} \text{ newton/coulomb}$$

where r is the distance of the point from the center, and q is the uniformly distributed charge on the sphere.

Potential at a point outside the charged sphere

$$V = -\int E\cdot dr$$

Substituting the value of E in above equation, we get

$$V = -\int \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\,dr$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}$$

Case 2: At a point on the surface of charged sphere, the electric field

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2} \text{ newton/coulomb}$$

where R is radius of the sphere.

On the surface of the charged sphere r = R. Therefore, electric potential

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R}$$

3. At a point inside the charged sphere, the electric field $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qr}{R^3}$

Now, we the electric potential will be

$$V = -\int E\,dr$$

$$= -\int \frac{1}{4\pi\varepsilon_0}\cdot\frac{qr}{R^3}\,dr$$

$$\text{or}\quad V = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{qr^2}{2R^3}$$

# 1.2 DIELECTRICS

*Dielectrics are the substances which do not contain free electrons or the number of such electrons is too low to constitute the electric current.* In dielectrics, the electrons are tightly bound to the nucleus of the atom. Mica, glass, plastic, etc. are examples of dielectrics. Faraday, first of all realized the importance of dielectrics in electrical phenomena. He introduced a slab of dielectric medium between the plates of a parallel plate condenser and found that the charge on the capacitor with dielectric is greater than that without. Thus, the capacitance of the capacitor increases because the charge is larger.

The dielectrics are used for

(i) maintaining two large metal plates at very small separation.

(i) increasing the potential difference So that a capacitor can withstand without breakdown.

(iii) increasing the capacitance of a capacitor.

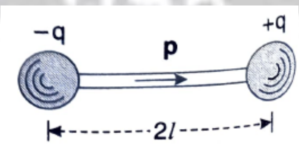
***Difference between dielectrics and conductors***

Following are the differences between dielectrics and conductors

| Dielectric | Conductor |
|---|---|
| (i) It is a material which does not contain free electrons. The electrons are tightly bound to the nuclei of atoms. | Conductors contain a fairly large number of free electrons. The free electrons wander through the conducting material |
| (ii) The dielectric does not conduct electricity. | Conductors conduct electricity |
| (iii) The charge given to a dielectric remains localized | The Charge resides on the surface in case of conductor |
| (iv) For a particular field strength (breakdown strength), the dielectric loses its insulation character. | Conductivity increases with field strength |

**ELECTRIC DIPOLE MOMENT**

*The arrangement of two equal and opposite point charges at a fixed distance is called an electric dipole. The product of the magnitude of either charge and the distance between the charges is called the electric dipole moment.*




Suppose the charges of dipole are - q and +q coulomb and the distance of separation is 2$l$ metre. Then the electric dipole moment p is given by

$$\mathbf{p} = q \times 2l = 2ql$$

The electric dipole moment is a vector quantity. Its direction is along the axis of dipole pointing to the negative charge to the positive charge. The unit of dipole moment is coulomb-metre. Here it should be remembered that the atom consisting of positive charges and negative charge not a dipole. The reason is that the centre of positive charges and the centre of negative charges coinside with each other, i.e., 2$l$=0.

But when the atom is placed in an electric field it becomes a dipole because the positive and negative centres are displaced relative to each other.

**TYPES OF POLARIZATION AND POLARIZABILITIES**

There are four different mechanisms by which electrical polarization can occur in dielectric materials when they are subjected to an external electric field. They are:

1. Electronic polarizatlon
2. Ionic polarization
3. Orientational polarization
4. Space charge polarization

1. Electronic polarizatlon

The electronic polarization occurs due to the displacement of positive and negative charges in dielectric material when an external electric field is applied. Figure (a) shows the charge distribution of an atom in the absence of the field while Fig. (b) shows the charge distribution in presence of applied electric field. This process occurs throughout the material and the material as a whole is polarized.

(a) Charge distribution in absence of field

(b) Charge distribution when field is applied

On the application of electric field, the displacement of positively charged nucleus and negatively charged electrons of the atom in opposite directions, results in electronic polarization.

As the nucleus and the centre of electron cloud are separated by a certain distance, dipole moment is created in each atom.

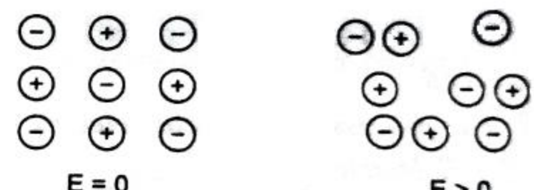
The induced dipole moment ($p_e$) is proportional to field strength (E), i.e. $p \propto E$.

If there are N atoms in the dielectric, then

$$P \propto N E \text{ or } P = \alpha_e N E$$

where $\alpha_e$ is electronic polarizability. The electronic polarizability is independent of temperature.

2. Ionic polarization



(a) Ion placement in absence of field

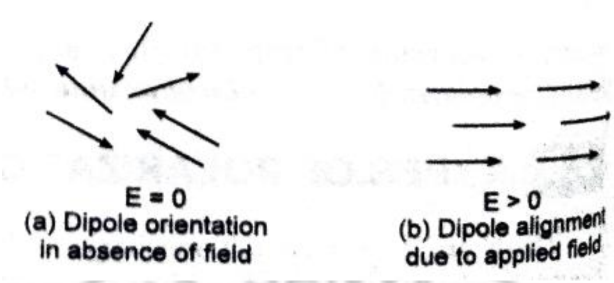
(b) Ion displacement in presence of field

Ionic polarization occurs only in those dielectric materials having ionic bonds such as NaCl. When such a material is subjected to an external electric field, the adjacent ions of opposite signs undergo displacement as shown in Fig (b). The displacement causes an increase or decrease in the distance separation between the atoms depending upon the location of ion pair. This leads to a net dipole moment. Therefore, the ionic polarization ($\alpha_i$) is due to displacement of cations and anions in opposite directions.

The polarizability in this case is known as ionic polarizability. This occurs in ionic solids. This polarization is also independent of temperature.

3. Orientational Polarization

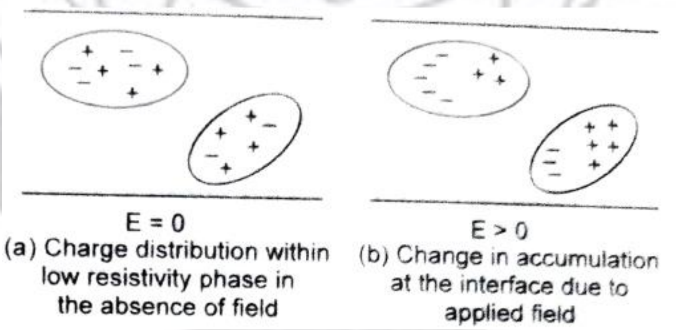
Polar dielectrics (such as $CH_3Cl$) exhibit orientational or dipolar polarizability. Even, in absence of external electric field, the polar dielectrics exhibit dipole moment. The orientations of the molecules are random and hence the net dipole moment is zero. The dipole orientation is shown in Fig. (a).




When an external field is applied to polar dielectrics, they tend to align themselves in the direction of external applied as shown in Fig. (b). The polarization due to such alignment is called orientation polarization ($\alpha_o$). The orientational polarization is strongly temperature dependent. This decreases with increase of temperature. The polarizability in this case is known as orientational polarizability.

4. Space-Charge Polarization

Space charge polarization occurs due to the accumulation of charges at the electrodes or at the interfaces in a multiphase materials as shown in Fig. The ions diffuse over appreciable distance in response to the applied field. This gives rise to redistribution of charges in the dielectric medium. The space-charge polarization is not an important factor in most common dielectrics.




Among the different polarizations, electronic and ionic polarizations are insensitive to temperature changes. The total polarization α of a material is sum of Electronic, ionic and orientational polarizations

i.e., $\alpha = \alpha_e + \alpha_i + \alpha_o$

**DIELECTRIC CONSTANT**

We have studied that when a dielectric is placed between the plates of a condenser its capacity is increased. The ratio of the capacitance of a condenser with dielectric to the capacitance of the same condenser without dielectric is defined as dielectric constant.

Thus, $$K = \frac{C\left(\text{Capacitance of the condenser with dielectric}\right)}{C_0\left(\text{Capacitance of the condenser without dielectric}\right)} \qquad (1)$$

Instead of maintaining the two capacitors at the same potential difference, we can give the same charge to each capacitor. It has been observed that the potential difference $V_d$ between the plates of the capacitor filled with dielectric is smaller than the potential difference $V_o$ without dielectric.

The ratio of potential difference without dielectric to the potential difference with dielectric is defined as dielectric constant.

Hence,

$$K = \frac{V_o}{V_d} \qquad (2)$$

According to Coulomb's law, the force of attraction or repulsion between two electric charges of magnitudes $q_1$ and $q_2$ separated by a distance $r$ in free space is given by

$$F_0 = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2}$$

when the charges are placed in some other medium, then

$$F = \frac{1}{4\pi\epsilon} \times \frac{q_1 q_2}{r^2}$$

Where $\epsilon$ and $\epsilon_0$ are the permittivity of the medium and permittivity of free space respectively.

$$\frac{F}{F_0} = \frac{\varepsilon_0}{\varepsilon} = \frac{1}{k}$$

$$k = \frac{\varepsilon}{\varepsilon_0} \qquad (3)$$

$$k = \frac{F_0}{F} \qquad (4)$$

From eq (3) the dielectric constant may be defined as the ratio of permittivity of the medium to permittivity of free space.

From eq (4), the dielectric constant may also be defined as the ratio of force between two charges in vacuum to the force between the same charges in dielectric medium.

Dielectric constant is a ratio. Its value is one for vacuum and $\infty$ for metals.

$$K = \frac{\text{Capacitance of the capacitor with dielectric}}{\text{Capacitance of the capacitor with air or vacuum ( without dielectric)}} = \frac{C}{C_0}$$

$$K = \frac{\text{Permittivity of the medium}}{\text{Permittivity of free space}} = \frac{\varepsilon}{\varepsilon_0}$$

$$K = \frac{\text{Force betweentwo charges in air}}{\text{Force between the charges in dielectric}} = \frac{F_0}{F}$$

$$K = \frac{\text{Potential difference without dielectric}}{\text{Potential difference with dielectric}} = \frac{V_0}{V_a}$$

The maximum electric field strength which a dielectric can withstand without breakdown called as dielectric strength of the medium.

## ELECTRIC SUSCEPTIBILITY

When a dielectric is placed in electric field it is polarized. The polarization vector P is proportional the electric field E. Hence,

$$P \propto E \qquad \text{or} \qquad P = \chi E$$

where the constant of proportionality $\chi$ is known as electric susceptibility. The electric susceptibility may be defined as the ratio of polarization vector to the electric intensity in the dielectric.

The electric susceptibility may also be defined as the ratio of induced surface charge density produced in the dielectric to the resultant electric field in the dielectric.

**Conclusion**

The electrical susceptibility is defined as follows:

$$\chi = \frac{\text{Polarization produced in the dielectric}}{\text{Electric intensity in the dielectric}} = \frac{P}{E}$$

$$\chi = \frac{\text{Induced surface charge density produced in dielectric}}{\text{Resultant electric field in the dielectric}} = \frac{\sigma_i}{E}$$

## RELATION BETWEEN DIELECTRIC CONSTANT AND SUSCEPTIBILITY

The polarization P is proportional to electric field E within the dielectric, i.e.,

$$P \propto E \quad \text{or} \quad P = \chi E$$

here $\chi$ is called as electric susceptibility of the medium.

We know that

$$D = \varepsilon_0 E + P$$

$$D = \varepsilon_0 E + \chi E \qquad (D = \varepsilon E)$$

$$\varepsilon = \varepsilon_0 + \chi$$

$$\frac{\varepsilon}{\varepsilon_0} = 1 + \frac{\chi}{\varepsilon_0}$$

$$k = 1 + \frac{\chi}{\varepsilon_0} = \varepsilon_r$$

$$\chi = (\varepsilon_r - 1)\,\varepsilon_0$$

This is the relation between susceptibility and dielectric constant.

## THREE ELECTRIC VECTORS AND THEIR RELATIONS

The three electric vectors are: (i) Electric intensity, (ii) Dielectric polarization, and (iii) Electric displacement.

1) Electric Intensity E:

The electric intensity E at any point in the electric field is numerically equal to the force experienced by a unit positive charge placed at that point. The direction of E being the same as that of the field.

(2) Dielectric polarization P:

When a dielectric is polarized, the distorted atom is called an electric dipole. The electric dipole moment per unit volume is called as dielectric polarization P.

(3) Electric displacement D

The electric displacement at a point is defined as the product of electric field strength (E) at that point and the permittivity of the medium ($\varepsilon$).

D = $\varepsilon$ E =k $\varepsilon_0$ E

In magnitude D is equal to the surface charge density ($\sigma$) of free charges

$$D = \frac{q}{A} = \sigma$$

**RELATION BETWEEN D, E and P**

When a dielectric slab is placed between the plates of a parallel plate condenser, the medium is polarized. Now, induced surface charges appear. The charge is negative on the surface nearer the positive plate of the condenser and positive charge nearer the negative plate. Let q' be the induced surface charge. Now the charge $q$ on the plate of the condenser and induced surface charge $q'$ are related as

$$\frac{q}{k\varepsilon_0 A} = \frac{q}{\varepsilon_0 A} - \frac{q'}{\varepsilon_0 A} \quad \ldots (1)$$

From eq. (1), we have

$$\frac{q}{\varepsilon_0 A} = \frac{q}{k\varepsilon_0 A} + \frac{q'}{\varepsilon_0 A}$$

or

$$\frac{q}{A} = \varepsilon_0 \left( \frac{q}{k\varepsilon_0 A} \right) + \frac{q'}{A} \quad \ldots (2)$$

We know that $\frac{q}{k\varepsilon_0 A} = E$ and $\frac{q'}{A} = P$

$\therefore$

$$\frac{q}{A} = \varepsilon_0 E + P$$

We put the ratio $\left(\frac{q}{A}\right)$ as $D$. Hence,

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \quad (3)$$

where D is called as electric displacement.

In terms of dielectric polarization P, a general form of the Gauss's law for E can be expressed as

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{(q - q')}{\varepsilon_0}$$

$$\varepsilon_0 \oint \mathbf{E} \cdot d\mathbf{S} = (q - q')$$

$$\varepsilon_0 \oint \mathbf{E} \cdot d\mathbf{S} = q - \oint \mathbf{P} \cdot d\mathbf{S}$$

$$\oint (\varepsilon_0 \mathbf{E} + \mathbf{P}) \cdot d\mathbf{S} = q$$

$$\oint \mathbf{D} \cdot d\mathbf{S} = q \quad \ldots (4)$$

Hence, dielectric displacement D is defined as a vector quantity whose surface integral charged surface over any (the flux of D), is equal to the free charge only within the surface.

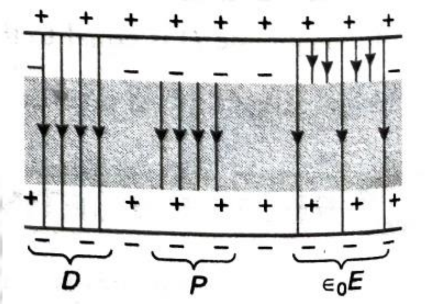
**Important Points**

The following points should be noted:

(i) D is connected with free charge only. It is not altered by the introduction of the dielectric. The lines of D begin and end on free charges

(ii) P is connected with polarization charge only. The lines of P begin and end on polarization charges.

(iii) E is connected with all charges that are free or actually present whether free or polarization. E is reduced inside the dielectric, where there are fewer lines.




**D and P in terms of E**

$$E = \frac{q}{\varepsilon_0 k A} \quad \text{or} \quad \frac{q}{A} = \varepsilon_0 k E$$

The vectors D and P are separately connected to E. We know that E

$$D = k\,\varepsilon_0 E \qquad \because \frac{q}{A} = D$$

The product k $\varepsilon_0$ is called the permittivity $\varepsilon$ of the medium and hence, D = $\varepsilon$ E. The ratio $\frac{\varepsilon}{\varepsilon_0}$ is also termed as relative permittivity because k is a ratio.
Putting the value of D in eq. (3), we get

$$k\,\varepsilon_0\,\mathbf{E} = \varepsilon_0\,\mathbf{E} + \mathbf{P}$$
$$\mathbf{P} = \varepsilon_0\,(k-1)\,\mathbf{E}$$

The constant $\varepsilon_0$ (k -1) in the above eq. is called as electric susceptibility χ of the dielectric. Hence,

$$P = \chi E$$

$\varepsilon_0$, k and χ are different ways of describing the property dielectric. These are related by relation:

$$\chi = \varepsilon_0\,(k-1) = \varepsilon_0\,k - \varepsilon_0 = \varepsilon - \varepsilon_0$$

$$\therefore \quad \varepsilon = \varepsilon_0 + \chi$$

Now, $$k = \frac{\varepsilon}{\varepsilon_0} = \frac{\varepsilon_0 + \chi}{\varepsilon_0} = 1 + \frac{\chi}{\varepsilon_0}$$

$$\therefore \quad \boxed{k = 1 + \frac{\chi}{\varepsilon_0}.} \qquad \ldots(8)$$

## BOUNDARY CONDITIONS AT THE DIELECTRIC SURFACE

The rules governing the behavior of E and D at the boundary between two dielectrics are known as boundary conditions.

Following are the two boundary conditions:

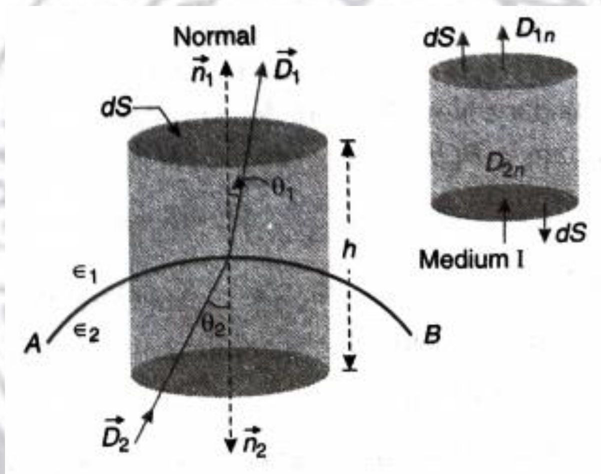
(i) The normal component of electric displacement D is the same on both sides of the boundary of two media of different dielectrics or the normal component of displacement vector is continuous across the charge free boundary between two dielectrics.

(ii) The tangential components of the electrical intensities are the same (continuous) on both sides of the dielectrics.

Derivation of Boundary Conditions

We shall derive these boundary conditions in the following way

1) Let AB represents a small portion of the boundary between two media of absolute permittivities $\varepsilon_1$ and $\varepsilon_2$ as shown in fig. The media are assumed to be homogeneous and isotropic.




A medium with same properties at every point is homogeneous and a medium with same properties in all directions is isotropic. We now consider a small area dS on the boundary so that its curvature may be neglected. Suppose $D_1$ and $D_2$ are the electric induction vectors in the media on either side of dS respectively. $\Theta_1$ and $\Theta_2$ are the angles which $D_1$ and $D_2$ make with the normal to dS. In order to find the boundary condition for D, let us construct a small pill box shaped surface which intersects the boundary with its end faces parallel to it. The height h of the pill box is assumed to be negligibly small in comparison with the diameter of the base.

Applying Gauss's theorem to the pill box surface, we have

$$\oint \mathbf{D} \cdot d\mathbf{S} = q \qquad \dots (1)$$

Let $D_{1n}$ be the average normal component of displacement-vector $D_1$ to the face of the pill box in medium I and $D_{2n}$, the average normal component of displacement vector $D_2$ to the face of the box in medium I. It should be remembered that $D_{2n}$ is along inward normal. Because the height of the pill box is negligibly small in comparison to the diameter of the base, the only contribution of flux of D will come through the end faces. Thus, by Gauss's law

$$D_{1n}\, dS - D_{2n}\, dS = q \qquad \dots (2)$$

where q is the total charge enclosed by the surface. The second term of eq. (2) is negative because $D_{2n}$ and dS oppositely directed.

From eq. (2)

$$D_{1n} = D_{2n} = \frac{q}{dS} = \sigma \quad \dots (3)$$

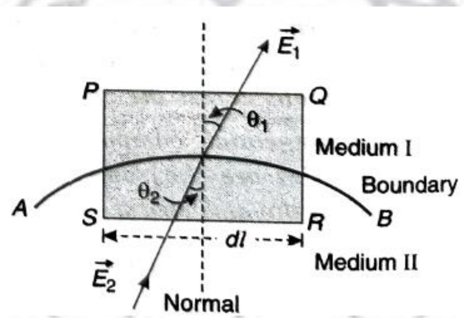
where $\sigma$ is the charge per unit area on the boundary of two dielectrics. Eq. (3) shows that the normal component of the displacement vector **D** changes at the charged boundary between two dielectrics by an amount equal to the surface charge density.

If the boundary is free from charge, then $\sigma = 0$. Now, eq. (3) reduces to

$$D_{1n} = D_{2n}. \quad \dots (4)$$

*Thus, the normal component of displacement vector is continuous across the charge free boundary between two dielectrics.*

(2) In order to consider the boundary condition for E, let us consider a rectangle PQRS of small width and its length parallel to the boundary separating two dielectrics as shown in fig.




The rectangle lies with its longest sides parallel to the boundary. Let $E_1$ and $E_2$ be the electric intensities in two media. The directions of the electric field vectors make angles $\Theta_1$ and $\Theta_2$ with the normal to the boundary. Now the work done in taking a unit charge around the rectangle PQRS must vanish, i.e.,

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0$$

$$E_1 \sin\theta_1 \, dl - E_2 \sin\theta_2 \, dl = 0. \quad \dots (5)$$

Here we have neglected the contribution of short sides QR and SP as they are taken very small. From eq. (5)

$$E_{1t} = E_{2t} \quad \dots (6)$$

Thus, the tangential component of the field is continuous at the boundary of two dielectrics.

# UNIT-II

## 2.1 ELECTRIC AND MAGNETIC FIELDS

**INTRODUCTION**

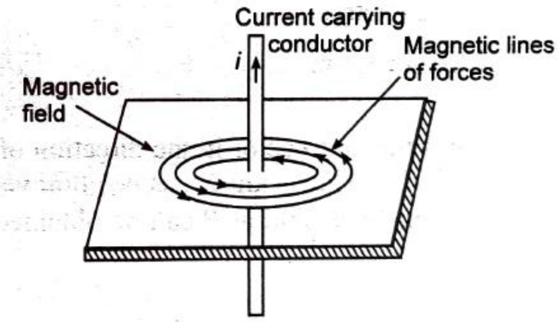
In 1820, ***Oersted*** discovered that a compass needle suffers a deflection when brought near a current carrying wire. This shows that an electric charge in motion produces a magnetic field in the space around it. It should be remembered that a charge (whether it is stationary or in motion) produces an electric field around it but when it is in motion, then in addition to *electric field it also produces a magnetic field*. So magnetism and electricity are two aspects of a single phenomenon. Now-a-days, electricity and magnetism are related by ***electromagnetism.***

**Origin of magnetism**

The earliest experiences with the magnetism involved ***Magnetite***, the only material that *occurs naturally in a magnetic state*. This mineral was also known as ***Lodestone***, after its property of aligning itself in certain directions if allowed to rotate freely, thus being able to indicate the positions of *North* and *South*, and to some extent also *latitude*. The other well-known property of Lodestone is that two pieces of it can attract or even repel each other.


**BIOT-SAVARTS LAW**

According to Oersted experiment, a current carrying conductor produces a magnetic field around it as shown in figure. The magnetic field exists as long as there is current in the conductor. French scientists Jean-Baptiste Biot and Félix Savart performed a series of experiments to study the magnetic field produced by various current carrying conductors in 1820. They obtained a relation to determine B at any point of space around a conductor that carrying current. The relation is called as Biot and Savart law.



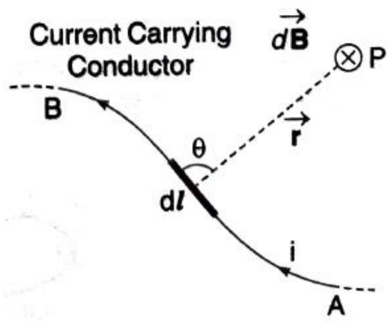

**Explanation:**

Let AB be a conductor of any arbitrary shape in which a current ***i*** is flowing. Let *P* be a point at which field is to be determined. According to Biot and Savart, the field B at any point can be computed by dividing the conductor into short current elements.

Let *dl* be the length of one such element and *r* be the displacement vector of point P from this element. According to Biot and Savart, magnetic field *d***B** due to small element d***l*** at P depend the following

(i) It is directly proportional to the current *i* flowing through the conductor

i.e., $dB \propto i$




(ii) It is directly proportional to the length d*l* of the element considered i.e., $dB \propto dl$

(iii) It is directly proportional to the sine of the angle θ between length of element and the line joining the element to the point P

i.e., $dB \propto \sin\theta$

(iv) It is inversely proportional to the square of the distance r of the point P from the element d*l*

i.e., $dB \propto \frac{1}{r^2}$

Combining all these factors

$$dB \propto \frac{i\,dl\,sin\theta}{r^2}$$

When the conductor is placed in vacuum or air, then

$$dB = \frac{\mu_0}{4\pi}\frac{i\,dl\,sin\theta}{r^2} \qquad (1)$$

where $\frac{\mu_0}{4\pi}$ is proportionality constant, $\mu_0$ is the permeability of free space.

In vector form,

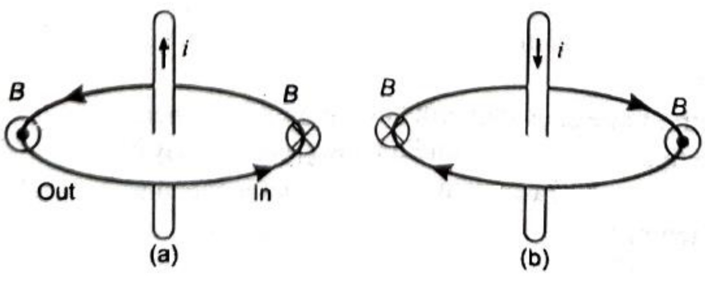
$$\boxed{d\bar{B} = \frac{\mu_0}{4\pi}\times\frac{i\,d\bar{l}\ \times\ \bar{r}}{r^3}}$$

where **r** is the unit vector in the direction of the line drawn from the current element dl to the point of observation P. This is known as position vector. The resultant field at P can be obtained by integrating eq. (1).

$$B = \int dB$$

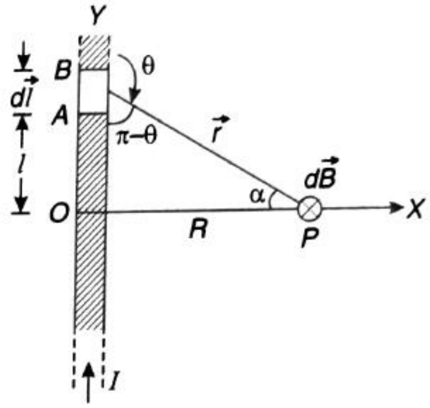
**DIRECTION OF B**

The direction of dB will be perpendicular to the plane containing dl and r. This is given by right hand rule. If magnetic field is directed perpendicular and into the plane of the paper, then it is represented by cross. When the magnetic field is directed perpendicular and out of the plane of the paper. it is represented by dot.



**MAGNETIC FIELD DUE TO LONG STRAIGHT CONDUCTOR (WIRE) CARRYING CURRENT**

Consider an infinitely long wire placed in vacuum and carrying a current i ampere as shown in fig. Let the distance of P from the conductor be R. O is the foot of the perpendicular from P to the conductor. Consider a small element AB of length *d***l** of wire at a distance from O. Let **r** be the distance of the element from the point P. Suppose θ be the angle in clockwise direction which the direction of current makes with the line joining the element to point P. According to Biot and Savart, magnetic field induction 'dB' at 'P' due to the small element 'AB' is given by




$$dB = \frac{\mu_0}{4\pi}\frac{i\,dl\,\sin\theta}{r^2} \qquad (1)$$

From figure,

$$r = \left(l^2 + R^2\right)^{\frac{1}{2}},$$

$$\sin\left(180 - \theta\right) = \sin\theta = \frac{R}{r}$$

Substituting these values in the above equation

$$\text{Now, } dB = \frac{\mu_0}{4\pi}\frac{i\,dl\,R}{\left(l^2+R^2\right)^{\frac{3}{2}}} \qquad (2)$$

The magnetic field induction 'B' at 'P' due to the entire conductor is

$$B = \int_{-\infty}^{\infty} dB = \int_{-\infty}^{\infty} \frac{\mu_0}{4\pi}\frac{i\,dl\,R}{\left(l^2+R^2\right)^{\frac{3}{2}}} \qquad (3)$$

In order to evaluate the integration substitute

$l$ = R tanα , $dl = R\sec^2\alpha\, d\alpha$

The limits of integration under this substitution become $-\frac{\pi}{2}$ *to* $\frac{\pi}{2}$ instead of -∞ to ∞

Then

$$B = \int_{-\pi/2}^{\pi/2} \frac{\mu_0}{4\pi}\frac{iR^2\sec^2\alpha}{\left(R^2\tan^2\alpha + R^2\right)^{3/2}}d\alpha$$

$$= \frac{\mu_0 i}{4\pi R}\int_{-\pi/2}^{\pi/2}\frac{\sec^2\alpha}{\left(1+\tan^2\alpha\right)^{3/2}}d\alpha = \frac{\mu_0 i}{4\pi R}\int_{-\pi/2}^{\pi/2}\frac{R^2\sec^2\alpha}{R^3\left(1+\tan^2\alpha\right)^{3/2}}d\alpha$$

$$= \frac{\mu_0 i}{4\pi R}\int_{-\pi/2}^{\pi/2}\frac{\sec^2\alpha}{\sec^3\alpha}\,d\alpha \qquad \left(\because \quad 1+\tan^2\alpha = \sec^2\alpha\right)$$

$$= \frac{\mu_0 i}{4\pi R}\int_{-\pi/2}^{\pi/2} \frac{1}{\sec\alpha}\, d\alpha \quad = \frac{\mu_0 i}{4\pi R}\int_{-\pi/2}^{\pi/2} \cos\alpha\, d\alpha$$

$$= \frac{\mu_0 i}{4\pi R}[\sin\alpha]_{-\pi/2}^{\pi/2}$$

$$= \frac{\mu_0 i}{4\pi R}\left[\sin\frac{\pi}{2} - \sin\left(\frac{-\pi}{2}\right)\right]$$

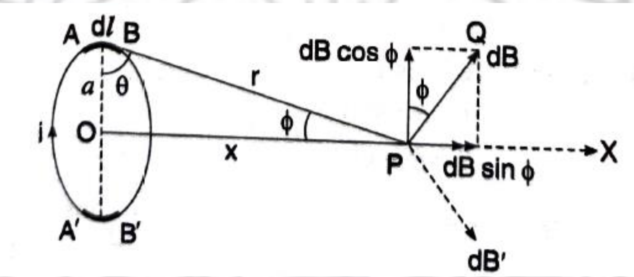
$$= \frac{\mu_0 i}{4\pi R}[1+1]$$

$$B = \frac{\mu_0 i}{2\pi R} \quad \text{web/m}^2 \quad \text{or tesla}$$

Hence, this is the expression for the magnetic field induction near a long straight conductor.

**MAGNETIC FIELD ON THE AXIS OF A CIRCULAR LOOP (COIL)**

Consider a circular loop of radius *a* and carrying a current *i*. P is a point on the axis of the coil at distant *x* from the centre. We are required to calculate the field at point P. Consider a small element AB of length ***dl***.




Let **r** be the distance of the element from the point P and θ be the angle which the direction of current makes with the line joining the element to the point O. The magnetic field d**B** at point P due to current element AB of length ***dl*** is given by

$$dB = \frac{\mu_0}{4\pi}\times\frac{i\,d\bar{l} \times \bar{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{\text{i dl sin}\theta}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{\text{i dl}}{r^2} \qquad (\because \theta = 90^0)$$

The vector d**B** at P is perpendicular to **r**. This can be resolved into two components d**B** cos ϕ and d**B** sin ϕ. Here d**B** cos ϕ at right angle to axis and dB sin ϕ along the axis of the coil.

If we take another element A'B' diametrically opposite to AB of the same length, it will also produce magnetic field dB' at P. The direction of d**B'** will be opposite to the previous one and equal in magnitude. This can also be resolved into components dB cos ϕ and dB sin ϕ. The components along the axis will add up while the components perpendicular to the axis will cancel. Similarly, if we divide the whole circular coil into a number of elements, the vertical components will cancel while the components along the axis will add up.

The magnetic field induction at P due to the circular coil is given by

$$B = \int dB \sin\phi$$

$$B = \int \frac{\mu_0}{4\pi}\frac{i\,dl}{r^2}\left(\frac{a}{r}\right) \qquad \left(\text{from fig., } \sin\phi = \frac{a}{r}\right)$$

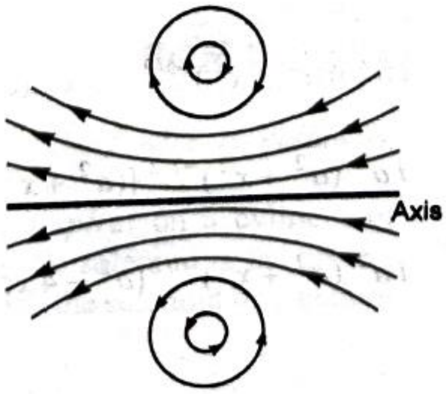
$$= \frac{\mu_0\, i\, a}{4\pi\, r^3}\int dl$$

$$= \frac{\mu_0 i\, a}{4\pi\, r^3}(2\pi\, a)$$

$$= \frac{\mu_0 i\, a^2}{2\, r^3}$$

From figure, $r = \left(a^2 + x^2\right)^{\frac{1}{2}}$

$$\therefore B = \frac{\mu_0 i\, a^2}{2\left(a^2 + x^2\right)^{3/2}}$$




If the coil has N turns then

$$B = \frac{\mu_0 i\, a^2\, N}{2\left(a^2 + x^2\right)^{3/2}} \text{ web/m}^2 \text{ or tesla}$$

The direction of B is along the axis of the coil as shown in fig.

**<u>Different Cases</u>**

(i) If point at the center of the coil i.e., $x = 0$.

Thus, Magnetic field at the center of the coil

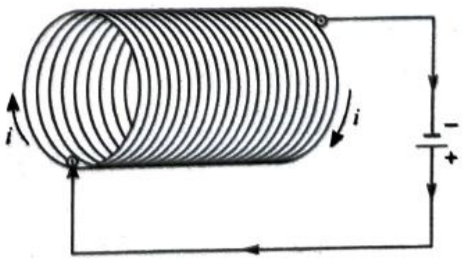
$$B = \frac{\mu_0 i\, N a^2}{2a^3} = \frac{\mu_0 N i}{2a}$$

(ii) At very far off from the loop $x >> a$ and $(a^2 + x^2)^{3/2} = x^3$ then

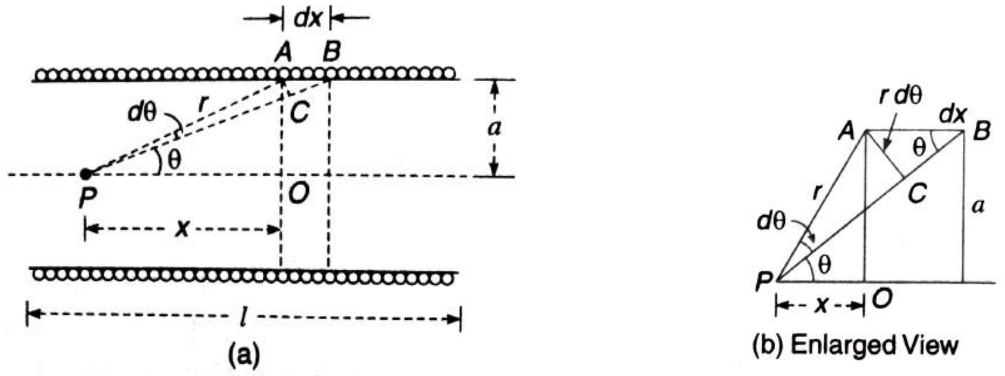
$$B = \frac{\mu_0 i\, N a^2}{2x^3}$$

## <u>MAGNETIC FIELD INDUCTION DUE TO A SOLENOID</u>

*A long, tightly wound helical coil of wire is called as solenoid.* Generally, the coil is wound as a spiral on a cylinder of non-magnetic material. The length of the solenoid is far greater than its diameter. The current is passed in the solenoid by a battery B as shown in the figure.




Consider a long solenoid of length $l$ metre and radius $a$ metre. Let N be the total number of turns. Then the number of turns per metre will be $n = N / l$. Suppose the solenoid carries current $i$ ampere.



(a) (b) Enlarged View

Now, we shall calculate the field in the following cases

**(i) Field at an inside point of the solenoid**

For this purpose we divide the solenoid into a number of narrow equidistant coils. We consider one small coil of width d$x$. Now the coil has n$dx$ turns. Let $x$ be the distance of point P from the centre O of the coil. The field at P due to elementary coil of width d$x$ carrying a current $i$ is given by

$$dB = \frac{\mu_0 i\, a^2\, n dx}{2\left(a^2 + x^2\right)^{3/2}} \text{ web/m}^2 \quad \text{or tesla} \qquad (1)$$

From figure, consider ΔABC, we have

$$\sin\theta = \frac{rd\theta}{dx} \Rightarrow dx = \frac{rd\theta}{\sin\theta}$$

From ΔAPO,

$$a^2 + x^2 = r^2 \Rightarrow \left(a^2 + x^2\right)^{3/2} = r^3$$

Substituting these values in eq. (1), we get

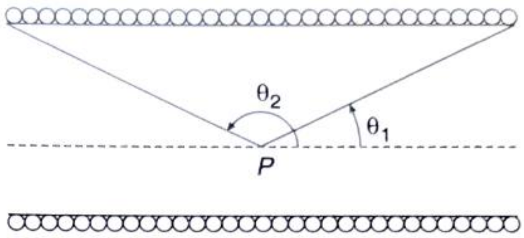
$$dB = \frac{\mu_0 i\, a^2\, n\left(\frac{rd\theta}{\sin\theta}\right)}{2r^3} = \frac{\mu_0 ni\, a^2 d\theta}{2r^2 \sin\theta}$$

$$= \frac{\mu_0 ni\, d\theta}{2\sin\theta}\left(\frac{a}{r}\right)^2$$

$$= \frac{\mu_0 ni\, d\theta}{2\sin\theta}\sin^2\theta$$

$$dB = \frac{\mu_0 ni\, d\theta \sin\theta}{2} \qquad (2)$$

The field induction B at P due to whole solenoid can be obtained by integrating the above eq. (2) between the limits $\theta_1$ and $\theta_2$. $\theta_1$ and $\theta_2$ are the semi-vertical angles subtended at P by first and last turns of the solenoid respectively.



The magnetic field induction B at P due to solenoid is

$$B = \int_{\theta_1}^{\theta_2} dB = \frac{\mu_0\ i\ n}{2}\int_{\theta_1}^{\theta_2} \sin\theta\ d\theta$$

$$= \frac{\mu_0\, i\, n}{2}\left[-\cos\theta\right]_{\theta_1}^{\theta_2} = \frac{\mu_0\ i\ n}{2}\left[-\cos\theta_2 + \cos\theta_1\right]$$

$$B = \frac{\mu_0\ i\ n}{2}\left[\cos\theta_1 - \cos\theta_2\right] \qquad (3)$$

(i) At any axial point P when it is well inside a very long solenoid, $\theta_1 = 0$ and $\theta_2 = \pi$.

Hence,

$$B = \frac{\mu_0\ i\ n}{2}\left[\cos 0 - \cos\pi\right]$$

$$= \frac{\mu_0\ i\ n}{2}[1-(-1)]$$

$$B = \mu_0 in \qquad (4)$$

(ii) Field at an axial end point

Now, $\theta_1 = 0$ and $\theta_2 = 90^o$

Hence,

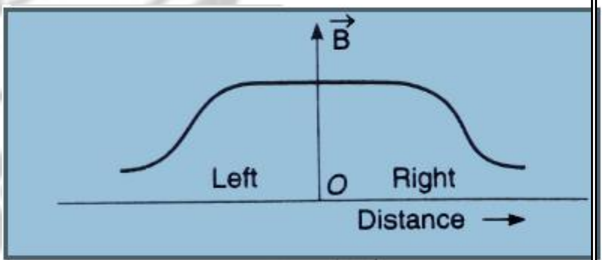
$$B = \frac{\mu_0\ i\ n}{2}\left[\cos 0 - \cos 90\right]$$

$$= \frac{\mu_0\ i\ n}{2}[1-0]$$

$$B = \frac{\mu_0 in}{2} \qquad (5)$$

This shovws that the field at either end is one half of its magnitude at the centre.

The variation of B with distance from the centre of a solenoid is shown in figure




**(ii) Field at the centre of a solenoid of finite length *l***

Consider that point P is at the centre, i e, it is a distance *l/2* from either end.
In this case,

$$\cos\theta_1 = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{\frac{l}{2}}{\left\{a^2 + \left(\frac{l}{2}\right)^2\right\}^{1/2}} = \frac{l}{\left(4a^2 + l^2\right)^{1/2}}$$

$$\cos\left(\pi-\theta_2\right) = \frac{\frac{l}{2}}{\left\{a^2+\left(\frac{l}{2}\right)^2\right\}^{1/2}} = \frac{l}{\left(4a^2+l^2\right)^{1/2}}$$

$$-\cos\theta_2 = \frac{l}{\left(4a^2+l^2\right)^{1/2}} \qquad \left[\because \cos\left(\pi-\theta_2\right) = -\cos\theta_2\right]$$

$$\cos\theta_2 = -\frac{l}{\left(4a^2+l^2\right)^{1/2}}$$

Putting these values in eq. (3), we get

$$B = \frac{\mu_0\ i\ n}{2}\left[\frac{l}{\left(4a^2+l^2\right)^{1/2}} + \frac{l}{\left(4a^2+l^2\right)^{1/2}}\right]$$

$$B = \frac{\mu_0\ i\ n}{2}\left[\frac{2l}{\left(4a^2+l^2\right)^{1/2}}\right]$$

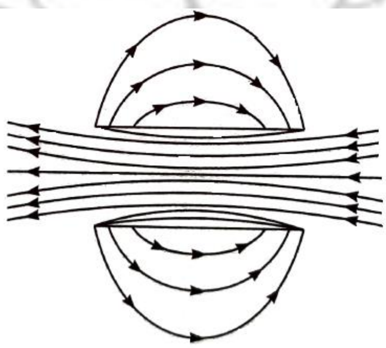
$$B = \frac{\mu_0\ i\ nl}{\left(4a^2+l^2\right)^{1/2}}$$

$$B = \frac{\mu_0\ i\ N}{\left(4a^2+l^2\right)^{1/2}} \qquad \left[\because \frac{N}{l} = n\right] \qquad (6)$$

This expression gives the field at the centre of the solenoid of finite length.

**Lines of Magnetic Induction**

The lines of magnetic induction inside and around the solenoid are shown as follows.



**LORENTZ FORCE**

1. When a charged particle having charge (q) moves in an electric field (E), tho things can be observed

   (i) If the charged particle moves in the direction of electric field, it experiences a force q E in tne direction of electric field. As a result, the charge particle is accelerated in the direction of electric field.

   (ii) If the charged particle moves in the opposite direction of electric field, it experiences a retarded force -q E. As a result, the charged particle is decelerated.

2. When a charged particle having charge q travels with velocity v is a magnetic field B, it experiences a force **F**= q(**v** × **B**) or F = qvB sin θ

Now,

(i) If the particle is rest in magnetic field (i.e., v = 0) then the particle will not experience any force.

(ii) If the particle is moving along the magnetic field, then v and B are parallel (i.e., θ =0). Hence, the particle will not experience any force.

(iii) If the particle is moving perpendicular to magnetic field, it experiences a maximum force denoted by $F_m$

The force on a charged particle moving in electromagnetic field (electric and magnetic field both) is known as Lorentz Force.

The force in electric field = q **E**

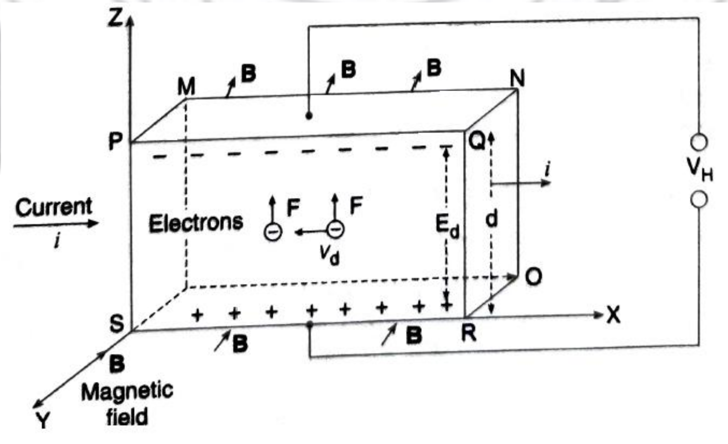
The force in magnetic field = q (v x B)

Therefore, Lorentz Force $\mathbf{F} = q\mathbf{E} + q(\mathbf{v}\times\mathbf{B}) = q[\mathbf{E} + (\mathbf{v}\times\mathbf{B})]$

## HALL EFFECT

Hall effect was discovered by E.H. Hall in 1879. *According to Hall Effect, when a magnetic field is applied perpendicular to a current carrying conductor, a potential difference is developed between the points on opposite side of the conductor*. This effect gives information about the sign of charged carriers in electric conductor.

Consider a uniform, thick metal strip placed with its length parallel to X-axis. Let a current i be passed in the conductor along X-axis and a magnetic field B be established along Y-axis. Due to the magnetic field, the charge carriers experience a force F perpendicular to X-Y plane, i.e., along Z-axis. The direction of this force is given by Fleming's left hand rule.




If the charge carriers are electrons, then they will experience a force in the positive direction of Z. Hence, they will be accumulated on the upper surface of the strip, i.e., on face PQNM as shown in fig. Due to this fact the upper side will be charged negatively while the lower side will be charged positively. Thus, a transverse potential difference is created. This e.m.f. is known as Hall e.m.f.

If the charge carriers are positively charged particles like protons or holes, the sign of e.m.f. is reversed. Thus, we can find the nature of charge carriers by determining the sign of Hall e.m.f. by a potentiometer. Experiments showed that the charge carriers in metals are electrons, while the charge carriers in P-type semiconductors are holes. Holes behave like positively charged particles. As discussed above, there is a displacement of charge carriers. This gives rise to a transverse field known as Hall electric field $E_H$. This field acts inside the conductor to oppose the sideway drift of the charge carrier.

**Hall Field and Hall Voltage**

When the equilibrium is reached, the magnetic deflecting forces on the charge carriers are balanced by the electric forces.

Magnetic deflecting force = q ($v_d$ x B)

Hall electric deflecting force = q $E_H$ ($E_H$ is Hall field)

As the net force on the charge carriers becomes zero

$$\mathbf{F} = q\mathbf{E}_H + q(\mathbf{v}_d \times \mathbf{B}) = q[\mathbf{E}_H + (\mathbf{v}_d \times \mathbf{B})] = 0$$

$$\mathbf{E}_H + (\mathbf{v}_d \times \mathbf{B}) = 0 \quad \text{or} \quad \mathbf{E}_H = -(\mathbf{v}_d \times \mathbf{B})$$

Writing in terms of magnitude only

$$E_H = -v_d\ B \qquad (1)$$

We know that drift velocity $v_d$ is related to the current density j by the following relation, i.e.,

$$v_d = \frac{j}{nq} \qquad (\because j = nqv_d) \qquad (2)$$

where n is the number of charge carriers per unit volume. Substituting in eq. (1), we get

$$E_H = \left(\frac{1}{nq}\right) jB \qquad (3)$$

If $V_H$ is the Hall voltage in equilibrium, then

$$E_H = \left(\frac{V_H}{d}\right) \qquad (4)$$

where d is the width of the bar.

**Hall Coefficient**

The ratio of Hall electric field $E_H$ to the product of current density j and magnetic induction B is known as Hall coefficient. This is denoted by $R_H$. So

$$R_H = \left(\frac{E_H}{jB}\right)$$

$$\frac{E_H}{jB} = \left(\frac{1}{nq}\right)$$

From eq.(3)

$$\therefore R_H = \left(\frac{1}{nq}\right)$$

Thus, measuring the potential difference $V_H$ between the two faces, $E_H$ can be calculated using eq. (4).

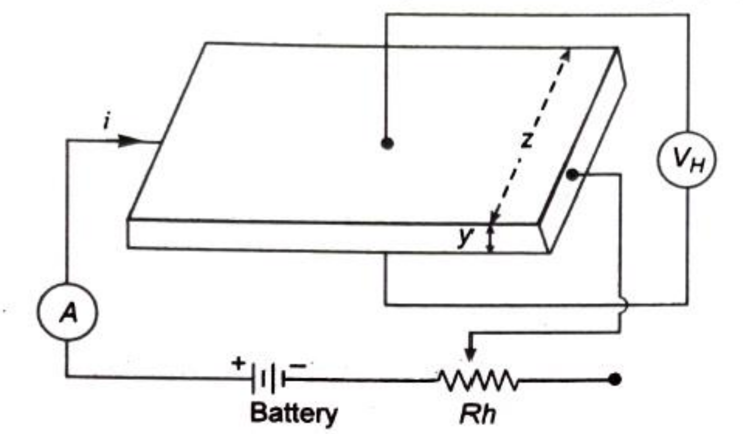
By measuring current *i* in the slab, the current density can be calculated by using (*i/A*), where A is the area of cross-section of the slab. The magnetic field B can be measured by a Gauss-meter. So, substituting the values of $E_H$ , j and B in eq. (3), we can calculate the value of $\left(\frac{1}{nq}\right)$.

The Hall coefficient is negative when the charge carriers are electrons and positive when charge carriers are holes.

**Determination of Hall Coefficients**

The arrangement for measurement of Hall voltage is shown in Fig.

A rectangular slab of given material is taken. A current (say *i*) is passed in this slab along X-direction with the help of battery and rheostat Rh. The slab is placed between the pole piece of an electromagnetic such that magnetic field is in a perpendicular direction of current (say in Y-direction). A voltmeter is connected in Z-direction as shown in figure. The voltmeter measures the Hall voltage $V_H$ . By measuring current *i* in the slab, the current density can be calculated by using (*i/A*), where A is the area of cross-section of the slab. The magnetic field B can be measured by a Gauss-meter. Hall electric field can be measured using formula $E_H = \left(\frac{V_H}{d}\right)$ . So, substituting the values of $E_H$ , j and B in eq. (3), we can determine the value of Hall coefficient $R_H = \left(\frac{E_H}{jB}\right)$




APPLICATIONS OF HALL EFFECT

Following are the applications of Hall effect:

(i) Hall effect gives information about the sign of charge carriers in electric conductor. It is found that most metals have negatively charged electrons.

(ii) Hall effect is quite helpful in understanding the electrical conduction in metals and semiconductors.

(iii) Hall effect can be used to measure the drift velocity of the charge carriers. We know that

$$v_d = \frac{j}{nq}$$

where n is the number of charge carriers per unit volume, j is current density and q is the charge on charge carrier.

(iv) Measurement of Hall coefficient gives the number of current carriers per unit volume. If n be the number of charge carriers per unit volume and A be the face area of plate, then

$$i = nqAv_d$$

$$v_d = \frac{E_H}{B}$$

$$\therefore \quad i = nqA\left(\frac{E_H}{B}\right)$$

Let L be the breadth and d the width of face area of conductor, then A = L d. So

$$i = nqLd\left(\frac{E_H}{B}\right)$$

$$n = \left(\frac{iB}{qLdE_H}\right)$$

For metals, the Hall coefficient is smaller than that for semi-conductors. The carrier concentration in semiconductor is much smaller. Hence, Hall effect devices use semiconductors rather the metals.

(v) The mobility of the caier can be measured by the conductivity of the material and Hall electric field

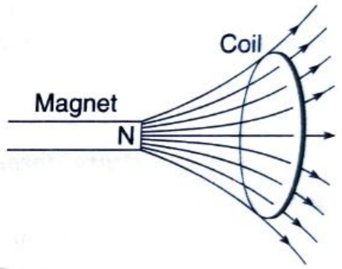
$$\mu = \sigma E_H$$

# 2.2 ELECTROMAGNETIC INDUCTION

The existence of the magnetic field due to the current flowing in a conductor was first discovered by Oersted. Ampere derived the expression for the intensity of the magnetic field. Faraday estimated the possibility of producing current from the magnetic field in 1831. He discovered that changing magnetic flux changing with time gives rise to current. It stands as long as the change lasts. The induced e.m.f. giving rise to such currents is called the induced electromotive force and the phenomenon is called electromagnetic induction.

**INDUCED EMF**

Let us consider the magnet and a coil experiment. When the magnet is moved towards the coil, the flux through the coil, increases. When the magnet is moved away from the coil, the flux through the coil decreases. In both the cases an induced e.m.f. is obtained in the coil during the motion of the magnet.




**<u>FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION</u>**

**<u>1st law:</u>** Whenever the magnetic flux $\phi_B$ linked with the coil changes an e.m.f is induced in it. This e.m.f exists as long as the flux changes. If the flux is constant, there is no induced e.m.f.

**<u>2nd law:</u>** The magnitude of the induced e.m.f is directly proportional to the negative rate of change of magnetic flux $\phi_B$.

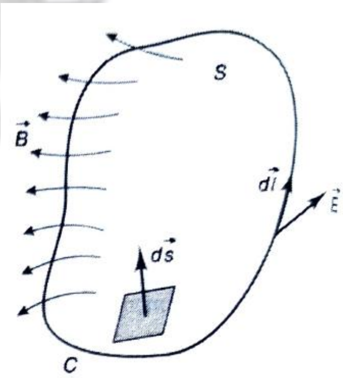
$$e = -\left(\frac{d\phi_B}{dt}\right) \qquad (1)$$

This law is also known as Neumann's law.

If there are N turns in the coil, then $e = -N\left(\frac{d\phi_B}{dt}\right)$

***<u>Integral and Differential forms</u>***

Consider a closed circuit C of any shape which encloses a surface S and magnetic field is produced by a stationary magnet or current carrying coil. Let B be the magnetic flux density in the neighborhood of the circuit. The magnetic flux through a small area dS will be B dS. Now the flux through the entire circuit is




$$\Phi_B = \int_S B.dS. \qquad (2)$$

When magnetic flux is changed, an electric field is induced around the circuit. The line integral of the electric field gives the induced e.m.f. in the closed circuit.

$$e = \oint E.dl \qquad (3)$$

Where E is the electric field at an element $d\mathbf{l}$ of the circuit. Substituting the values of $e$ and $\phi_B$ from eqs. (3) and (2) in eq. (1), we have

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}.$$

This is the integral form of Faraday's law.

The line integral of the electric field around any closed circuit is equal to the negative rate of change of magnetic flux through the circuit.

By Stokes theorem, $$\oint \mathbf{E} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S} \quad \ldots(5)$$

From eqs. (4) and (5), we get

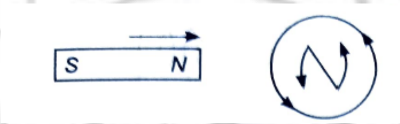
$$\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

From the above equation $\nabla \times \mathbf{E} = -\frac{\partial B}{\partial t}$ or $\text{curl } \mathbf{E} = -\left(\frac{\partial \mathbf{B}}{\partial t}\right)$

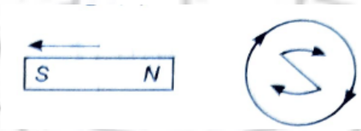
This is the differential form of Faraday's law

Faraday's law gives the correct direction of induced e.m.f. or current. Let us consider that north pole is approaching the coil. The magnetic flux is increasing in a direction away from the observer. In this case, (dØ/dt) is negative i.e., (-(dØ/dt)) is positive. Hence, this gives rise an e.m.f. and current in anticlockwise direction.

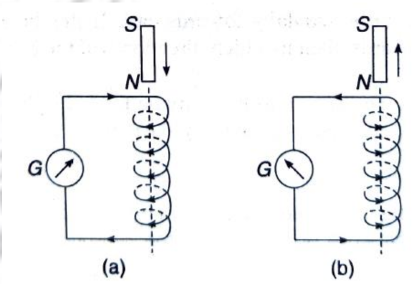



On the other hand, when the north pole is moved away from the coil, then (dØ/dt) is positive i.e.,

(-(dØ/dt)) is negative. This gives rise to a clockwise e.m.f. and current in the coil.

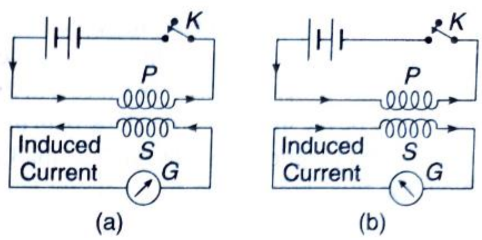



**Example 1:** Consider a coil of wire connected in series with a galvanometer. When a magnet is inserted in the coil, the galvanometer shows a deflection in one direction and when it is withdrawn from the coil, the galvanometer shows the deflection in the other direction. This indicates a momentary current in the coil. When the magnet is stationary, there is no deflection in the galvanometer.




If the experiment is repeated with other pole of magnet facing the coil, deflections are reversed. It is further observed that when the magnet is moved fast, the deflection in the galvanometer is large when it is moved slowly, the deflection is small, i.e, the deflection depends upon the rate at which magnet is inserted or withdrawn.

**Example 2:** Consider a circuit in which a primary coil P connected to a battery and secondary coil S connected to a galvanometer. When the circuit is closed by pressing the key K and then broken, the galvanometer connected in the secondary shows a deflection first in one direction and then in the other direction. It is observed that no deflection is produced in the galvanometer if the current in the primary circuit flows continuously. The deflection is produced in the galvanometer only at make and break.

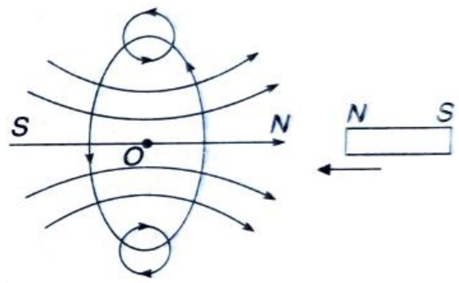


Similar effects are observed while increasing or decreasing the primary current or changing the relative position of the coils.

***Lenz's Law:***

This law gives the direction of the induced e.m.f. The induced e.m.f (or current) will always act in a direction such that it opposes the cause which has produced it.

The law is based on the principle of conservation of energy. Thus, when the applied flux density B in a closed circuit is increasing, the e.m.f. or current induced in the closed circuit is in such a direction as to produce a field which tends to decrease B.




On the other hand, when the applied flux density is decreasing in magnitude the current in the closed circuit is in such a direction as to produce a field which tends to increase B.

Thus, the induced current is in a direction such that it produces a magnetic flux tending to oppose the original change of flux, i.e, tending to keep the total flux constant in the circuit. Suppose the north pole of the magnet is moved towards a coil connected to a galvanometer as shown in fig. As the magnet is pushed towards the circuit, an induced current is set up in the coil.

The induced current produces its own magnetic field. Now the coil behaves as a magnet. The face of the coil towards the north pole of the magnet becomes a north pole. So, there will be a force of repulsion between them. Due to this force of repulsion, the motion of the magnet is opposed. This causes a change of magnetic flux in the coil. Thus, the direction of induced current is such that it opposes the motion of the magnet.

**Conclusion**

When the north pole N is approaching the coil, then magnetic flux is increasing through the coil i.e., induced e.m.f. and current are set up in the coil.

According to Lenz's law, the induced current should be such that the motion of north pole towards the coil is opposed. This is possible when there arises a north pole on the face of coil. As a result, the Current flows in anticlockwise direction.

**SELF INDUCTION**

The phenomenon of self induction was discovered by J. Henry (American) in 1832. When a current flows in a coil, magnetic field is set up in it. If the current in the coil changes with time, an induced e.m.f. is set up in the coil. According to Lenz's law, the direction of induced e.m.f. is such as to oppose the change in current. When the current increasing, the induced e.m.f. is against the current and when the current is decreasing it is in the direction of current. So the induced e.m.f. opposes any change of the original current. The phenomenon is called self induction.

*The phenomenon of production of an induced emf in a circuit itself due to variation of current through the same circuit (increasing or decreasing) is called self-inductance.*

*The property of the circuit by virtue of which any change in the magnetic flux linked with it, induces an e.m.f. in it, is called inductance and the induced e.m.f. is called back e.m.f.*

When the current in a coil is switched on, self induction opposes the growth of the current, and when current is switched off, the self induction opposes the decay of current.

**Coefficient of Self Induction or Self Induction (L)**

The total magnetic flux D is proportional to the current i, flowing through it, i.e.

$$\Phi_B \propto i$$

$$\Phi_B = L\,i \quad \text{or} \quad L = \frac{\Phi_B}{i}\left(\frac{\text{weber}}{\text{ampere}}\right)$$

where L is a constant called the coefficient of self induction or self inductance of the coil.

Hence, the coefficient of self induction is numerically equal to the magnetic flux linked with the coil when unit current flows through it.

The e.m.f. induced in the coil is given by

$$e = -\frac{d\Phi_B}{dt} = -\frac{d(L\,i)}{dt} \qquad (\text{because } \Phi_B = L\,i)$$

$$e = -L\frac{di}{dt}. \qquad \ldots(2)$$

The negative sign indicates that the induced e.m.f. is in such a direction as to oppose the change the coefficient of self-inductance is numerically equal to the induced e.mf. in the course when the rate of change of current is unity.

$$\frac{di}{dt} = 1, \qquad e = -L$$

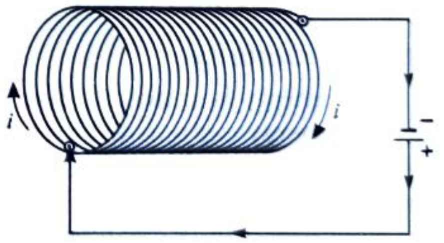
The unit of self-inductance is henry

$$1\text{ henry} = \frac{\text{volt}}{\text{ampere/second}} = \frac{1\text{ volt}}{1\text{ ampere per second}}.$$

$$1\text{ mH} = 10^{-3}\text{ henry} \quad \text{and} \quad 1\,\mu\text{H} = 10^{-6}\text{ henry}.$$

**INDUCTANCE OF LONG SOLENOID**

Consider a long air core soleniod (of small diameter) of length and uniform cross-section area/metre A metre. Let n be the number of turns per metre. Suppose a current i amp. flows through it. The magnetic field inside the solenoid is given by




$$B = \mu_0 ni \text{ weber/metre}$$

Magnetic flux through each turn

$$\phi_B = B * A = \mu_0 niA \text{ weber}$$

Now the magnetic flux linked with all the turns of solenoid

$$\phi_B = \mu_0 niAN \text{ weber turn}$$

where N is equal to total number of turns in the solenoid.

$$= \mu_0\, n\, i\, A \times n\, l \qquad (\because\ N = n\, l)$$

$$= \mu_0\, n^2\, i\, A\, l$$

The self inductance of the solenoid is therefore.

Li=Total flux linked with the solenoid

$$L\,i = \mu_0\, n^2\, i\, A\, l$$

$$L = \mu_0\, n^2\, A\, l \ \text{henry}$$

where n is number of turns per unit length.

In terms of total number of turns N of the solenoid

$$L = \mu_0 \left(\frac{N}{l}\right)^2 A\, l$$

$$L = (\mu_0\, N^2\, A)/l \ \text{henry}$$

The self inductance L depends upon:

(i) Length of the coil (*l*)

(ii) Number of turns in the coil (N)

(iii) Cross-sectional area of the coil (A).

**ENERGY STORED IN MAGNETIC FIELD**

Consider a very long solenoid of length *l* and cross-sectional area A. When a current flows in it, magnetic field is established. This field is uniform inside and negligible outside. So the volume associated with the magnetic field is A *l*.

We have seen that the amount of work done in establishing a current i in the solenoid is (L i)

where L is the inductance of solenoid. The work done is stored as energy in the magnetic field.

$$U = \text{energy stored} = \frac{1}{2} L\, i_0^{\,2}$$

The inductance of the solenoid is given by

$$L = \mu_0\, n^2\, A\, l,$$

where n is number of turns in solenoid per metre

$$U = \frac{1}{2}(\mu_0\, n^2\, A\, l)\, i_0^{\,2} = \frac{1}{2}\frac{(\mu_0\, n\, i_0)^2}{\mu_0} A\, l$$

The magnetic field inside the solenoid

$$= B = \mu_0\, n\, i_0$$

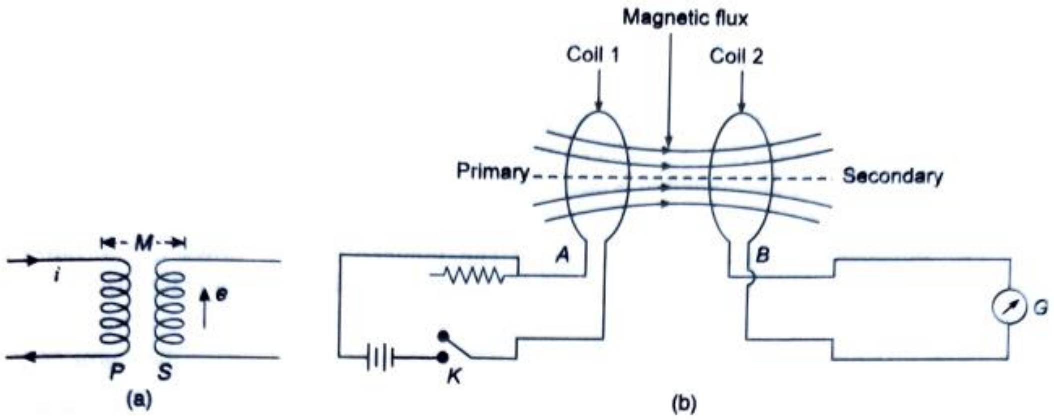
$$U = \frac{1}{2}\cdot\frac{B^2}{\mu_0} A\, l$$

So the energy density (energy per unit volume) u in magnetic field is given by

$$u = \frac{U}{A\, l} = \frac{1}{2}\cdot\frac{B^2}{\mu_0}\cdot\frac{A\, l}{A\, l} = \frac{1}{2}\frac{B^2}{\mu_0} \ \text{joule/metre}^3$$

$$u = \frac{B^2}{2\mu_0} \ \frac{\text{joule}}{\text{metre}^3}$$

## COEFFICIENT OF MUTUAL INDUCTION




Consider two coils placed near to each other as shown in fig. When a current is passed in the primary coil P, there is a change of magnetic flux linked with it, and an induced e.m.f. is set in the secondary coil S. This phenomenon is called mutual inductance. Just as the primary circuit produces an induced e.m.f. in the secondary, similarly the secondary circuit also induces an e.m.f. in the primary. Hence, the total induction through the secondary during the time the induced current lasts in it, is the difference between the inductions due to primary and secondary Any two circuits in which there is mutual induction are known as mutually coupled circuit. Leta current i amp. in primary Produces a magnetic flux D in the secondary S. For two given coils situated in fixed relative positions, it is observed that the flux linked with the secondary is proportional to the current in primary. Thus,

$$\Phi_B \propto i$$
$$\Phi_B = M\, i$$

where M is a constant called the "Coefficient of Mutual induction" or "Mutual inductance" of the two coils. The e.m.f. induced in the secondry S is given by

$$e = -\frac{d\Phi_B}{dt}$$
$$e = -\frac{d}{dt} M\, i = -M \frac{di}{dt}$$

The egs. (1) and (2) enable us to define the mutual inductance in the following two ways: 1. fis the flux linked with a clrcult due to u unit current Jlowing through the other. 2. Itis the emf, induced in the cireui, when the rate of adecay of eurrent in the other cireuit is unig: The unit of mutual inductance is henry. It is the mutual inductance of two circuits when the current changing at the rate of one amp/sec in one circuit induces an e.m.f. of I volt in the other eireuit. Important: Let 2 be the flux through coil 2 due to a current 4 in coil 1, then

$$\Phi_{21} = M_{21}\, i_1 \qquad \Phi \propto i_1$$

where $M_{21}$ is called as coefficient of mutual induction or mutual inductance of coil 2 due to coil 1. Let 2 be the flux through coil 1 due to a current in coil 2, then

$$\Phi_{12} = M_{12}\, i_2$$

where $M_2$ is called as coefficient of mutual inductance of coil 1 due to coil 2.

**Distinction between Self induction and Mutual induction**

| S.No | Self induction | Mutual induction |
|---|---|---|
| 1 | When the current flowing in a coil is changed, an induced current is prodced in the col itself. This is called as self-induction. | When the current flowing in a coil is changed, an induced current is prodced in the col itself. This is called as self-induction. |
| 2 | The induced current affect the main current. | 2. The induced current flows in the other coil. It does not affect main current of primary coil |
| 3 | There is only one coil in it | There are 2 miorr |

## MUTUAL INDUCTANCE OF TWO GIVEN COILS

Consider a long air cored solenoid with primary A and secondary B as shown in fig. (12).

Let number of turns in the primary =n

length of primary coil =*l*

area of cross-section = a

number of turns in secondary = n2 current in the primary = *i*

Magnetic field inside the primary =4oiweber/metre2

Magnetic fux through each turn of primary

$$\Phi = B\ A = \mu_0 \frac{n_1}{l} i \times a \text{ weber}$$

Since secondary is wound closely over the central position of primary and hence, the same flux i also linked with each turn of the secondary. . Total magnetic flux linked with secondary

$$= \mu_0 \frac{n_1\ i}{l} \times a \times n_2 \text{ weber turn}$$

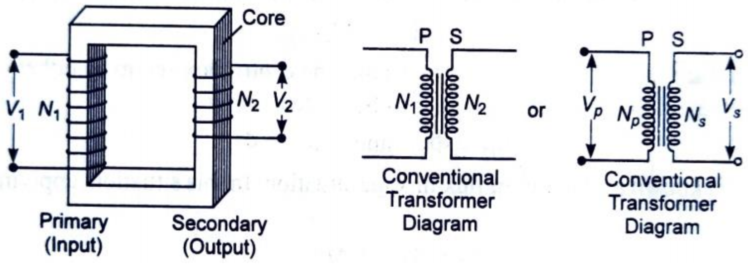
If M be mutual inductance of the two coils, the total flux linked with the secondary is M i

$$M\ i = \mu_0 \frac{n_1\ i}{l} \times a \times n_2$$

$$M = \frac{\mu_0\ n_1\ n_2\ a}{l}.$$

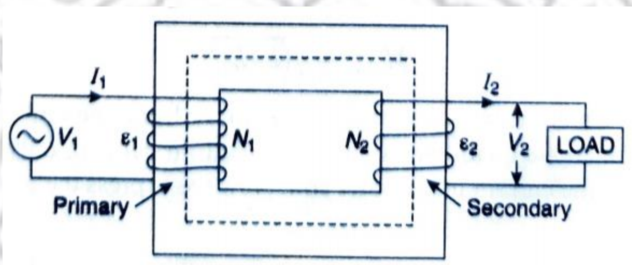
## TRANSFORMER

Construction a transformer is an A.C. static device which transfers electric power from one circuit to another. i can raise or lower the voltage in a circuit but with a corresponding decrease or increase in current. Here it is worth mentioning that while transferring the power, frequency is not altered. A transformer is a device which allows a voltage change in an A.C. supply voltage with small on of power Fig. shows a basic transformer.




### Construction

The transformer consists of two coils. One is known primary coil (P) while the other is knowns secondary coil (S). The two separate coils (primary and secondary) are wound on the same ferromagnetic core as shown in fig. Due to magnetic core, the mutual inductance between the two coils will be minimum. The two coils are electrically insulated but they are connected magnetically. The energy from one coil is transferred to other coil by means of magnetic coupling. The coil which receives energy from an A.C. source is called primary (P) and the coil which delivers the energy to load is called secondary (S) as shown in fig.




The number of turns in the primary are expressed by N, while the number of turns in secondary are expressed by $N_2$. The ratio $N_2/N_1$ is called as Turns ratio or Transformer ratio. This is represented by a. When the number of turns in secondary ($N_2$) is more than the number of turns in primary ($N_1$) i.e., $N_2 > N_1$ then the transformer is known as step-up transformer. When the number of turns is primary ($N_1$) is more than the number of turns in the secondary ($N_2$) i.e., $N_1 > N_2$, then the transformer is known as step-down transformer.

### Principle

A transformer operates on the principle of mutual induction. When an alternating voltage is applied to the primary, an alternating current is set up in it. As the winding is linked with a magnetic e.m.f., it produces an alternating flux in the core. This alternating flux links with the turns of the secondary coil. Since the flux is alternating in magnitude, it induces a mutually induced e.m.f. in secondary of the same frequency as the flux. This follows from Faraday's law of electromagnetic induction,

i.e., $e = M\,(dI/dt)$

Because of this induced e.m.f, the secondary coil is capable of supplying current and hence energy. Consider that no load is connected in the secondary i.e., secondary is open. Now an a.c. current $i_1$ will flow through primary coil. This causes a magnetic flux through primary. This induces an e.m.f. equal and opposite to$V_1$. Let $\Phi$ be the flux through the core. We have,

$$V_1 = \varepsilon_1 = -N_1 \frac{d\Phi}{dt} \qquad (1)$$

The same flux is linked with the secondary coil. Therefore, the secondary voltage $V_2$ is given by

$$V_2 = \varepsilon_2 = -N_2 \frac{d\Phi}{dt} \qquad \textbf{(2)}$$

Dividing eq. (2) by eq. (1), we get

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = a \quad \text{or} \quad \frac{V_1}{N_1} = \frac{V_2}{N_2}$$

where a is turn ratío or transformer ratio.

When N2> N1, then V2>V1, or a>1, the transformer is known as step up transformer.

When N1 > N2, then V1>V2 or a<1, the transformer is known as step down transformer.

For an ideal transformer, there is no loss of energy and we have

$$V_1 i_1 = V_2 i_2$$

$$\frac{i_1}{i_2} = \frac{V_2}{V_1} = a$$

This equation shows that when there is a voltage increase (step-up transformer) the output current ($i_2$) will decrease. On the other hand, when there is a voltage decrease, (step-down transformer), the output current $i_2$ will increase

**Step-up Transformer and Step-down Transformer**

If N> N, then a > 1, transformer is called step-up transformer If N2> N1, then a<1 , transformer is called step-down transformer. For an ideal transformer

Input V x A = Output V x A

$$V_1 i_1 = V_2 i_2$$

$$\frac{V_1}{V_2} = \frac{i_2}{i_1}.$$

# UNIT-III

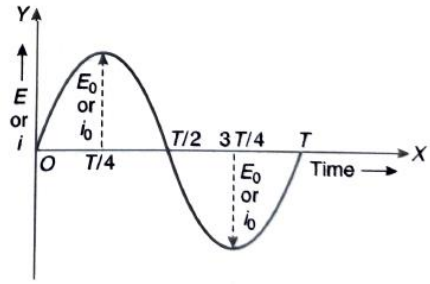
## ALTERNATING CURRENT

An alternating current or a.c. is defined as one which passes through a cycle of changes at regular intervals. The waveform of such a voltage or current is shown in fig. (1) and is mathematically represented by

$$i = i_o \, sin\omega t$$

$$E = E_o \, sin\omega t$$

Here i or E represents the instantaneous value whereas the peak or maximum value is represented by $i_o$ or $E_o$. The term $\omega t$ is called the phase. The time of one cycle is known as time period represented by T and the number of cycles per second gives the frequency of supply (f =1/T).




One cycle of alternating current consists of two half cycles during one of which the current is entirely positive whereas during the next half-cycle it is entirely negative. So the d.c. meter will indicate zero deflection as it measures the average value of complete cycle. Hence, the term root-mean square or effective value is used to measure the AC. Current i(t) in a pure resistor R results in a power p(t) with an average value P. This same P could be produced in resistance R by a constant current I. Then i(t) is said to have an effective value $i_{rms}$ equivalent to this constant current I. The same can be applicable to Voltage functions where the effective value is $E_{rms}$.

Hence, a.c. instruments measure the square root of the mean square value or virtual value. Therefore, when we say supply voltage in our homes is 230 volt, we are talking about the rms value. The peak voltage in this case is $\sqrt{2}$ time the rms value.

The average value of the rectified current is the same as the average current in any half cycle. It is $2/\pi$ times the maximum current $i_o$. Then the d.c. meter can be calibrated accordingly to measure a.c.

**Average Value of A.C. during Complete Cycle**

Now, we shall mathematically prove that the average or mean value of a.c. in one complete cycle zero. The value of current at any instant t is given by

$$i = i_o \, sin\omega t$$

The average value of a sinusoidal wave over one complete cycle is given by

$$i_{av} = \frac{\int_0^T i_o \, sin\omega t \, dt}{\int_0^T dt} = \frac{-\frac{i_o}{\omega}|\cos \omega t|_0^T}{T}$$

$$= -\frac{i_o}{\omega T}\left|\cos\frac{2\pi t}{T}\right|_0^T$$

$$= -\frac{i_o}{\omega T}[\cos 2\pi - \cos 0]$$

$$= -\frac{i_o}{\omega T}[1 - 1] = 0$$

Thus, we see the average value of a.c. over one complete cycle is zero. Similarly, we can prove for alternating e.m.f.

**Mean or Average Value of A.C. Voltage for Half Cycle**

The mean or average of A.C. is the average of the sum of the instantaneous values taken for half a cycle (i.e., for half a period). Now the sum of the instantaneous values for half a cycle is given by

$$\int_0^{T/2} i\, dt = \int_0^{T/2} i_o \sin\omega t\, dt$$

Mean value of current is given by

$$i_{avg} = \frac{\int_0^{T/2} i_o \sin \omega t\, dt}{\int_0^{T/2} dt} = \frac{2}{T}\int_0^{T/2} i_o \sin \omega t\, dt$$

$$= \frac{-2i_0}{T\omega}\left[\cos \omega t\right]_0^{T/2}$$

$$= \frac{-2i_0}{T\frac{2\pi}{T}}\left[\cos\frac{2\pi}{T}.\frac{T}{2} - \cos 0\right]$$

$$i_{avg} = \bar{i} = \frac{-i_0}{\pi}[-1-1] = \frac{2i_0}{\pi}$$

$$i_{avg} = \frac{2}{\pi} i_o$$

Similarly $E_{avg} = \frac{2}{\pi} E_o$

**RMS Value of A.C. Voltage or current**

Root mean square value for complete cycle can be calculated as follows.

$$i_{eff}^2 R = \overline{i^2} R$$

$$i_{eff} = \sqrt{\overline{i^2}} = i_{RMS}$$

$$\overline{i^2} = \frac{\int_0^T i_o^2 \sin^2 \omega t\, dt}{\int_0^T dt}$$

$$= \frac{i_0{}^2}{T}\int_0^T \sin^2 \omega t\, dt$$

$$= \frac{i_0{}^2}{T}\int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt = \frac{i_0{}^2}{2T} = \frac{i_0{}^2}{2T}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T = \frac{i_0{}^2}{2\,T}\left[T - \frac{\sin 2\left(\frac{2\pi}{T}\right)T}{2\left(\frac{2\pi}{T}\right)} - 0 + \frac{\sin 2\left(\frac{2\pi}{T}\right)(0)}{2\left(\frac{2\pi}{T}\right)}\right]$$

$$\overline{i^2} = \frac{i_0{}^2}{2T}\left[T - \frac{\sin 4\pi}{\left(\frac{4\pi}{T}\right)}\right] = \frac{i_o^2}{2\,T}\left[T - 0\right] = \frac{i_o^2}{2}$$

$$i_{eff} = i_{RMS} = \sqrt{\overline{i^2}}$$

$$i_{RMS} = \sqrt{\frac{i_o^2}{2}} = \frac{i_o}{\sqrt{2}}$$

Similarly, $E_{RMS} = \frac{E_o}{\sqrt{2}}$

**AC Through pure Resistance:**

Consider a circuit containing only pure resistance 'R' and alternating e.m.f. E is applied.

$E = E_0 \sin\omega t$ ----- (1)

From Ohms law E = iR

$E_0 \sin \omega t = iR$

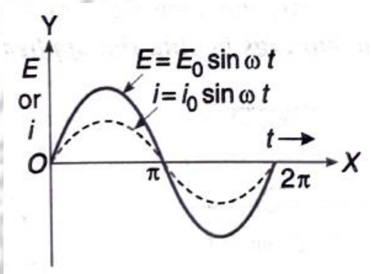
$i = (E_o / R) \sin \omega t$ ------ (2)

i will be maximum when the term $\sin \omega t$ is unity

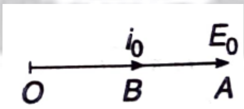
$i = i_o \sin \omega t$ ------ (3)

where $i_o = E_o / R$

From (1) and (3) it is clear that voltage and current are in phase. Phase diagram is shown in figure.



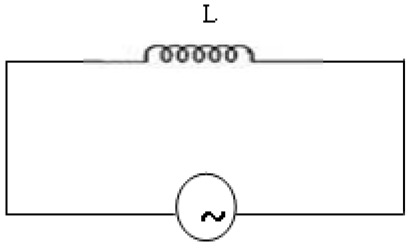

Voltage and current vectors for a pure resistance circuit have been shown in figure. OA is the voltage vector and OB is the current vector




**AC Through pure Inductance:**

Suppose that an alternating e.m.f. is applied to an AC circuit containing an inductor L. Due to self-inductance of coil an induced e.m.f. (L $\frac{di}{dt}$) is generated opposite to the change in current through it.

Now the instantaneous current 'i' is derived as




$E = E_0 \sin\omega t$

$$E = E_0 \sin\omega t \qquad \text{----- (1)}$$

$$\text{Induced emf} = L\,\frac{di}{dt}$$

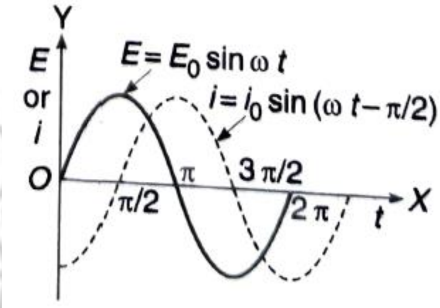
$$\therefore E_0 \sin\omega t = L\,\frac{di}{dt}$$

$$\text{But } di = \frac{E_0}{L}\sin\omega t\, dt \quad => \quad i = \int \frac{E_0}{L}\sin\omega t\, dt$$

$$\text{On integrating } \; i = \frac{E_0}{\omega L}(-\cos\omega t)$$

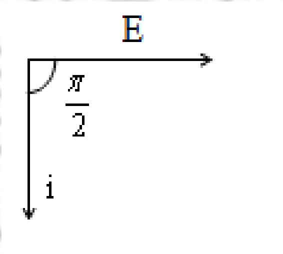
$$i = \frac{E_0}{X_L}\left[\sin\left(\omega t - \frac{\pi}{2}\right)\right]$$

$$i = i_o \sin(\omega t - \pi/2) \qquad \text{----- (2)}$$




Here $X_L = \omega L$ is called inductive reactance. Its S.I unit is ohm ( Ω )

From equation of voltage and current, current lags behind the voltage by

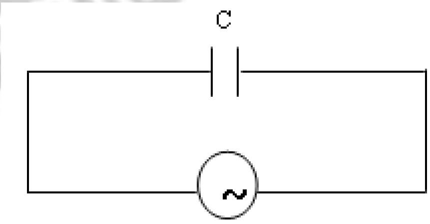



π/2 radian or 90°. Hence, following is the vector-diagram by taking the voltage as reference.

$X_L = \omega L = 2\pi f L$ ohm, $X_L$ depends directly on frequency of the applied voltage. The higher the value of $f$, the greater is the reactance offered and vice-versa.

**A.C. THROUGH PURE CAPACITANCE ONLY**

Let us apply an alternating voltage $E = E_0 \sin\omega t$ to a pure capacitor of capacitance C as shown in




$E = E_0 \sin\omega t$

fig. Then the capacitor will get charged first in one direction and then in the opposite direction. If q

is the charge on plates at any instant, then

$$q = CE \qquad \text{-------- (1)}$$

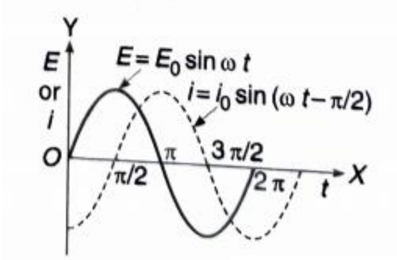
$$\text{but} \quad E = E_0 \sin\omega t \qquad \text{-------- (2)}$$

Putting value of E, we get $q = C E_0 \sin\omega t$

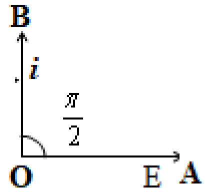
Differentiate the above equation

$$\frac{dq}{dt} = E_0 \omega C \cos\omega t$$

$$i = \frac{E_0}{1/\omega C}\left[\sin\left(\omega t + \frac{\pi}{2}\right)\right] \qquad \left[\because \frac{dq}{dt} = i\right]$$




$$i = \frac{E_0}{X_C}\left[\sin\left(\omega t + \frac{\pi}{2}\right)\right]$$

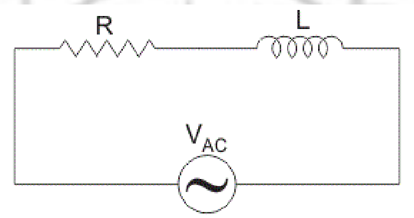



Here $X_C = \frac{1}{\omega C}$ is called capacitive reactance. Its S.I unit is ohm ($\Omega$).

$$\therefore\ i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \text{-------- (3)}$$

From (2) and (3) it is clear that current through the capacitor leads the voltage by $\frac{\pi}{2}$. Phaser diagram is shown in the figure.

**<u>AC circuit containing Resistance and Inductance (RL or LR circuit):</u>**




Consider a circuit containing inductance 'L' and a resistance R in series with an alternating e.m.f $E = E_0 \sin\omega t$. Let i be the instantaneous value of the current in the circuit. Due to the changing current, an induced e.m.f. $[-L(di/dt)]$ is set up in the inductance which opposes the applied e.m.f.

Now the effective e.m.f. in the circuit = $E_0\ \sin\ \omega t\ - L(di/dt)$

According to Ohm's law, this must be equal to Ri. Hence,

$$E_0\ \sin\ \omega t\ - L(di/dt) = iR$$

$$E_0\ \sin\ \omega t\ =\ iR + L\left(\frac{di}{dt}\right)$$

$$L\left(\frac{di}{dt}\right) + iR = E_0\ \sin\ \omega t \qquad \text{------ (1)}$$

We know that

The trial solution of eq (1) is

$$i\ =\ i_0 \sin\ (\omega t - \phi) \qquad \text{------ (2)}$$

$$\frac{di}{dt} = i_0\omega\ \cos(\omega t - \phi)$$

Substitute these values in equation (1)

$$E_0 \; sin\;(\omega t - \phi + \phi) = R\; i_0\; sin(\omega t - \phi) \;+\; L\; i_0 \omega\; cos(\omega t - \phi)$$

$$E_0\left[sin(\omega t - \phi)\; cos\phi \;+\; cos(\omega t - \phi)\; sin\phi\right] \;=\; R\; i_0 sin(\omega t - \phi) \;+\; L\; i_0 \omega\; cos(\omega t - \phi)$$

Comparing the coefficients of sin(ωt-Ø), cos(ωt-Ø)

$$E_0\; cos\phi \;=\; Ri_0 \text{ ------ (3)}$$

$$E_0\; sin\phi \;=\; X_L i_0 \text{ ---- (4)}$$

$$(3)^2 + (4)^2 \Rightarrow E_0^{\;2} = \{R^2 + X_L^{\;2}\}\; i_0^{\;2}$$

$$\Rightarrow i_0^{\;2} = \frac{E_0^{\;2}}{R^2 + X_L^{\;2}}$$

$$\Rightarrow i_0 = \frac{E_0}{\sqrt{R^2 + (X_L)^2}}$$

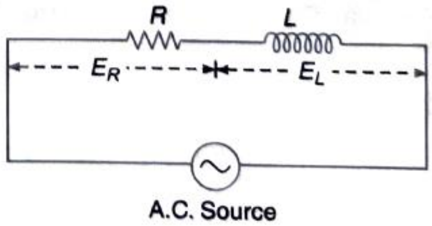
But $\sqrt{R^2 + X_L^{\;2}}$ = Z is called impedance of circuit. Its S.I unit is ohm ( $\Omega$ ).

$$\therefore i = \frac{E_0}{Z} \text{ ------- (5)}$$

$$\frac{(3)}{(4)} \Rightarrow \tan\phi \;=\; \frac{X_L}{R}$$

$$\phi \;=\; \tan^{-1}\left(\frac{X_L}{R}\right)$$

**VECTOR DIAGRAM OF RL OR LR CIRCUIT**




Consider a shows a circuit containing resistance (R) and inductance (L) connected to an alternating Source. The current in all parts will be the same. Let $\mathbf{E_R}$ and $\mathbf{E_L}$ be voltages across the resistance (R) and inductance (L) respectively.

We know that

(i) voltage across resistance always remain in phase with the current.

(ii) The voltage across inductance lead over current by 90°.

Hence,
$$E_R = iR$$
$$and\;\; E_L = i\omega L = iX_L$$

$E_R$ may be represented along current line of the circuit while $E_L$ at 90° ahead to $E_R$ as shown in fig. These are indicated by vectors OA and OB respectively.

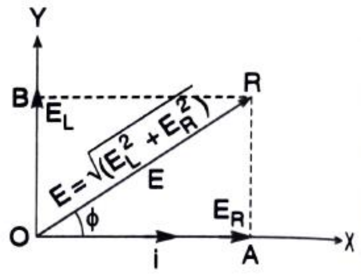
By the law of parallelogram of vector addition, the diagonal OR represents the impressed voltage E across the inductance and resistance in series. Hence,

$$E^2 = E_R^2 + E_L^2$$

$$(iZ)^2 = (iR)^2 + (i\omega L)^2$$

$$Z^2 = R^2 + \omega^2 L^2$$

$$Z = \sqrt{R^2 + \omega^2 L^2}$$



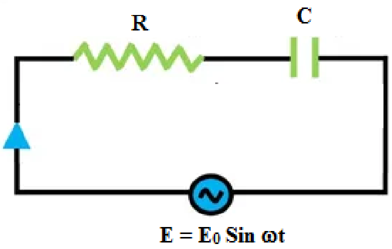

Where Z is the impedance of the circuit.

Further,
$$\tan\ \phi = \frac{E_L}{E_R} = \frac{iX_L}{iR} = \frac{i\omega L}{iR} = \frac{\omega L}{R}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

**<u>AC circuit containing Resistance and Capacitance (RC or CR circuit):</u>**

Consider a capacitor of capacitance 'c' and resistance R connected in series with an alternating e.m.f $E = E_0 \sin\omega t$.




Consider a circuit containing a resistance R and a capacitance C in series, connected to alternating e.m.f. source ($E = E_0 \sin \omega t$) as shown in fig. Let q be the charge on the capacitor at any instant t and i be the current in the circuit at that instant.

The potential difference across the capacitor at this instant is $\frac{q}{C}$

This opposes the applied e.m.f., So the effective e.m.f. in the circuit will be $E_0\ sin\ \omega t\ -\ \frac{q}{C}$

According to Ohm's law, this must be equal to Ri.

Hence, $E_0\ sin\ \omega t\ -\ \frac{q}{C} = \text{R}\ i$

$$\text{R}\ i + \frac{q}{C} = E_0\ sin\ \omega t \qquad \text{------ (1)}$$

Differentiating this equation, we get

$$\text{R}\ \frac{di}{dt} + \frac{i}{C} = E_0\omega\ \cos\omega t \qquad \left[\because \frac{dq}{dt} = i\right] \qquad \text{------ (2)}$$

The solution of eq. (2) is of the form

$$i\ =\ i_0\ sin\ (\omega t - \phi) \qquad \text{------ (3)}$$

where i and o are constants to be determined

Then, $$\frac{di}{dt} = i_0\omega\ \cos(\omega t - \phi)$$

Substitute these values in eq. (2), we get

$$R\, i_0\omega\, \cos(\omega t - \phi) + (1/C)\, i_0\, \sin(\omega t - \phi) = E_0\omega\, \cos\omega t = E_0\omega\, \cos((\omega t - \phi) + \phi)$$

$$R\, i_0\omega\, \cos(\omega t - \phi) + (1/C)\, i_0\, \sin(\omega t - \phi) = E_0\omega\left[cos(\omega t - \phi)\, cos\phi - sin(\omega t - \phi)\, sin\phi\right]$$

Comparing the coefficients of $\sin(\omega t - \phi)$ and $\cos(\omega t - \phi)$ in the above equation, we get

$$(1/C)\, i_0 = -E_0\omega\, sin\phi$$

$$\Rightarrow \quad -(1/\omega C)\, i_0 = E_0\, sin\phi \qquad \text{------ (4)}$$

$$R\, i_0\omega = E_0\omega\, cos\phi$$

$$\Rightarrow \quad R\, i_0 = E_0\, cos\phi \qquad \text{------- (5)}$$

By doing $(4)^2 + (5)^2 \Rightarrow \quad E_0^{\,2} = \left\{R^2 + (1/\omega C)^2\right\} i_0^{\,2}$

$$\Rightarrow \quad i_0^{\,2} = \frac{E_0^{\,2}}{R^2 + (1/\omega C)^2}$$

$$i_0 = \frac{E_0}{\sqrt{R^2 + (1/\omega C)^2}} = \frac{E_0}{\sqrt{R^2 + X_C^{\,2}}} \qquad \text{------ (6)}$$

$\therefore$ Impedance $Z = \dfrac{E_0}{i_0} = \sqrt{R^2 + X_C^{\,2}} = \sqrt{R^2 + (1/\omega C)^2}$

Z is called impedance of circuit. Its S.I unit is ohm ($\Omega$).

By doing (4) ÷ (5) $\Rightarrow \quad \tan\phi = \dfrac{(1/\omega C)}{R}$

$$\Rightarrow \quad \phi = \tan^{-1}\left\{\frac{(1/\omega C)}{R}\right\}$$

$$\therefore \quad \phi = \tan^{-1}\left\{\frac{X_C}{R}\right\}$$

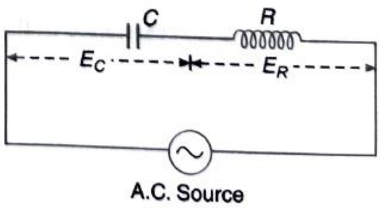
Substituting $i_0$ in eq. (3) we get

$$i = \frac{E_0}{\sqrt{R^2 + (1/\omega C)^2}}\, sin\, (\omega t - \phi)$$

where $\phi = \tan^{-1}\left\{\dfrac{X_C}{R}\right\}$

**VECTOR DIAGRAM OF RC OR CR CIRCUIT**

Consider a shows a circuit containing resistance (R) and capacitance (C) connected to an alternating Source. The current in all parts will be the same. Let $\mathbf{E_R}$ and $\mathbf{E_C}$ be voltages across the resistance (R) and capacitance (C) respectively.



We know that

(i) Voltage across resistance always remain in phase with the current.

(ii) The voltage across the condenser lags behind the current by 90°.

Hence, $E_R = iR$

$and\ E_C = i/\omega C = iX_C$

$E_R$ may be represented along current line of the circuit while $E_C$ at 90° below to $E_R$ as shown in figure. These are indicated by vectors OA and OB respectively.

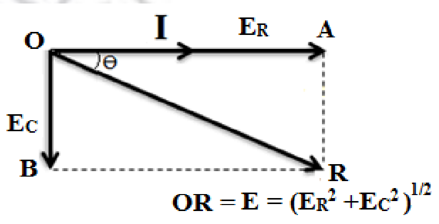
By the law of parallelogram of vector addition, the diagonal OR represents the impressed voltage E across the capacitance and resistance in series. Hence,

$$E^2 = E_R^2 + E_C^2$$

$$(iZ)^2 = (iR)^2 + (i/\omega C)^2$$

$$Z^2 = R^2 + (1/\omega^2 C^2)$$

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = \sqrt{R^2 + X_C^{\ 2}}$$



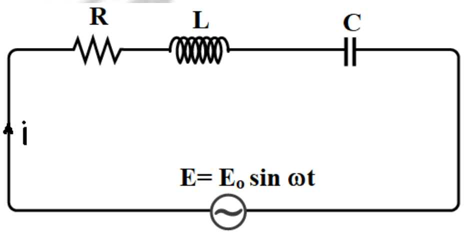

Where Z is the impedance of the circuit.

Further, $$\tan\ \phi = \frac{E_C}{E_R} = \frac{iX_C}{iR} = \frac{i/\omega C}{iR} = \frac{1}{\omega CR}$$

$$\phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

**Series L-C-R Circuit:**

Consider a capacitor of capacitance C, Inductor of inductance L and resistance R are connected in series with an alternating e.m.f. $E = E_0 \sin\omega t$.




Let i = instantaneous current in circuit and

q = instantaneous charge on the capacitor.

The potential difference across capacitor is q/C and back e.m.f. induced in the inductance is $L\frac{di}{dt}$.

The effective e.m.f. in the circuit is $E_0 sin\omega t - \frac{q}{C} - L\frac{di}{dt}$

According to Ohm's law, this must be equal to Ri

$$E_0 sin\omega t - \frac{q}{C} - L\frac{di}{dt} = R\ i$$

$$L\frac{di}{dt} + R\ i + \frac{q}{C} = E_0 sin\omega t \qquad \text{------ (1)}$$

Differentiate above equation

$$L\frac{d^2i}{dt^2}+R\ \frac{di}{dt}+(1/C)i=\omega E_0\cos\omega t \qquad \left[\because \frac{dq}{dt}=i\right] \qquad \text{------ (2)}$$

The solution of eq. (2) is of the form

$$i\ =\ i_0\ sin\ (\omega t-\phi) \qquad \text{------ (3)}$$

Then, $$\frac{di}{dt}=i_0\omega\ \cos(\omega t - \phi) \qquad \text{------ (4)}$$

$$\frac{d^2i}{dt^2}=-i_0\omega^2\ \sin(\omega t - \phi) \qquad \text{------ (5)}$$

Substitute the above values in eq. (2), we get

$$-L\ i_0\omega^2 sin(\omega t-\phi)+\ R\ i_0\omega\ cos(\omega t-\phi)+(1/C)\ i_0 sin(\omega t-\phi)=\omega E_0\cos\omega t$$

$$-L\ i_0\omega^2 sin(\omega t-\phi)+\ R\ i_0\omega\ cos(\omega t-\phi)+(1/C)\ i_0 sin(\omega t-\phi)=\omega E_0\cos((\omega t-\phi)+\phi)$$

$$=\omega E_0\left[cos(\omega t-\phi)cos\phi\ -\ sin(\omega t-\phi)sin\phi\right]$$

Comparing the coefficients of $\sin(\omega t-\phi)$ and $\cos(\omega t - \phi)$ in the above equation, we get

$$-L\ i_0\omega^2+(1/C)\ i_0=-\omega E_0\ sin\phi$$

$$\Rightarrow \quad L\ i_0\omega-(1/\omega C)\ i_0=E_0\ sin\phi \qquad \text{------ (6)}$$

$$R\ i_0\omega=\omega E_0\ \cos\phi$$

$$\Rightarrow \quad R\ i_0=E_0\ \cos\phi \qquad \text{------ (7)}$$

By doing $(6)^2 + (7)^2 \Rightarrow$ $$E_0{}^2=\left\{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2\right\}i_0{}^2$$

$$\Rightarrow \quad i_0{}^2=\frac{E_0{}^2}{\left\{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2\right\}}$$

$$\therefore \quad i_0=\frac{E_0}{\sqrt{\left\{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2\right\}}} \qquad \text{------ (8)}$$

By doing (6) ÷ (7) $\Rightarrow$ $$\tan\phi=\frac{\left(\omega L-\frac{1}{\omega C}\right)i_o}{Ri_o}$$

$$\tan\phi=\frac{\left(\omega L-\frac{1}{\omega C}\right)}{R}$$

$$\therefore \quad \phi = \tan^{-1}\left\{\frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}\right\} \qquad \text{------ (9)}$$

From eq. (8), $\frac{E_0}{i_0} = \sqrt{\left\{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right\}} = \sqrt{\left\{R^2 + \left(X_L - X_C\right)^2\right\}} = Z$

Here $Z = \frac{E_0}{i_0} = \sqrt{\left\{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right\}}$ is the impedance of the circuit and has SI units ohms.

Substituting Eq. (8) in eq. (3), we get

$$i = \frac{E_0}{\sqrt{\left\{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right\}}} sin\ (\omega t - \phi)$$

The current lags in phase from the e.m.f. by an angle $\phi$

Here $\phi$ is the phase difference between voltage 'E' and current 'i'.

**Case i:** if $\omega L > \frac{1}{\omega C}$ then $\phi$ is positive.

i.e., the current in the circuit lags behind the voltage by $\phi$.

**Case ii:** if $\omega L < \frac{1}{\omega C}$ then $\phi$ is negative.

i.e., the current in the circuit is leads the voltage by $\phi$.

**Case iii:** if $\omega L = \frac{1}{\omega C}$ then $\phi$ is zero.

i.e., the current in the circuit is phase with the voltage.

**<u>SERRIES RESONANCE CIRCUIT</u>**

Maximum current $i_0$ from LCR Series circuit is $i_0 = \frac{E_0}{\sqrt{\left\{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right\}}}$

And the impedance Z of the circuit is given by $Z = \sqrt{\left\{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right\}}$

The L-C-R series circuit has a very large capacitive reactance (l/ωC) at low frequencies and large inductive a verv resistance (ωL) at high frequencies. So at a particular frequency, the total reactance in the circuit is zero (ωL = l/ωC). Under this situation, the resultant impedance of the circuit is a minimum (equal to R).

The particular frequency of A.C. at which impedance of a series L-C-R circuit becomes minimum (or the current becomes maximum), [when ωL = l/ωC] is called the resonant frequency and the circuit is called as series resonant circuit.

At resonant frequency $X_L = X_C$

$$=> \omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$=> \omega = \frac{1}{\sqrt{LC}}$$

We know that $\omega = 2\pi f_0 = \frac{1}{\sqrt{LC}}$

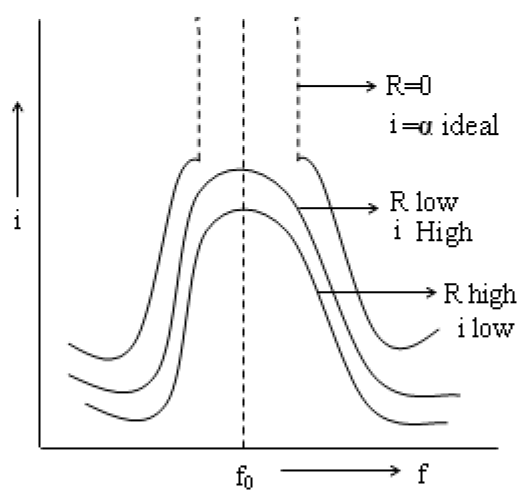
So, the resonant frequency $f_0$ of the series resonant circuit is given by $f_0 = \frac{1}{2\pi\sqrt{LC}}$

The resonant frequency depends on the product of L and C and does not depend on R.

The L-C-R series circuit at this frequency is called series resonance crcuit.

The variation of the current peak value with the frequency of the applied e.m.f. is as shown in the fig. If the resistance is low, then the curve is sharp. It is called sharpness of resonance. For higher resistance values the peak losses its sharpness.

Observations:

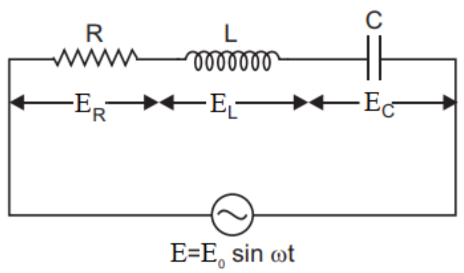



(i)The maximum current occurs at a particular frequency called as resonant frequency

(ii) The peak of the curve depends on the resistance of the circuit. When R is low, the peak is high and vice-versa. The peak is known as sharpness of resonance. More is height of peak, sharper is the resonance.

(iii) The series resonant circuit is called as acceptor circuit. Because out of all currents, it accepts the current whose frequency is same as the resonance frequency only.

**VECTOR DIAGRAM OF SERIES LCR CIRCUIT**

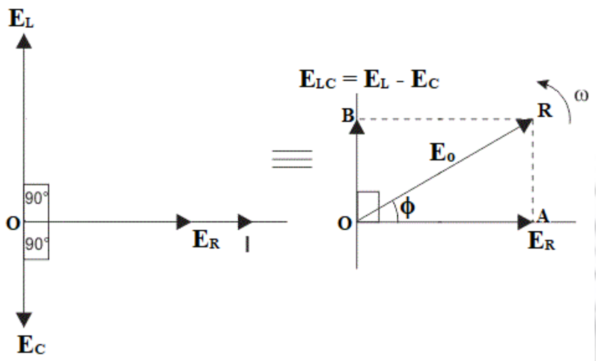



Consider an AC circuit containing an inductance (L), capacitance (C) and resistance (R) in series.

Let $E_L$, $E_C$ and $E_R$ be the potential differences across inductance, capacitance and resistance respectively. $i_o$ is the peak value (rms value) of current in the circuit. The voltage and current in the resistor will be in the same phase at all times. The voltage across the inductance $E_L = i_o\ \omega L$, will lead the current by $\pi/2$ (90°) while the capacitive reactance $E_C = i_o\ (1/\omega C)$ will lag behind the current by $\pi/2$ (90°).

Therefore, voltage amplitude (OA) and current amplitude on resistor are expressed on x-axis in vector diagram. Here $E_L$ and $E_C$ are in antiphase, expressed on y-axis. The resultant of the two is represented by $E_{LC} = E_L - E_C = i_o\ \{\omega L - (1/\omega C)\}$.

The resultant of $E_R$ and $E_{LC}$ (OB) can be obtained by vector addition method. This is represented by OR.




Thus,

$$E_0^{\ 2} = (OR)^2 = \left(Ri_o\right)^2 + \left[i_o\left(\omega L - \frac{1}{\omega C}\right)\right]^2 = i_o^{\ 2}\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]$$

$$E_0 = (OR) = i_o\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{\frac{1}{2}} = \sqrt{\left\{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right\}}$$

We now that impedance $Z = \dfrac{E_0}{i_0}$

So, $$Z = \sqrt{\left\{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right\}}$$

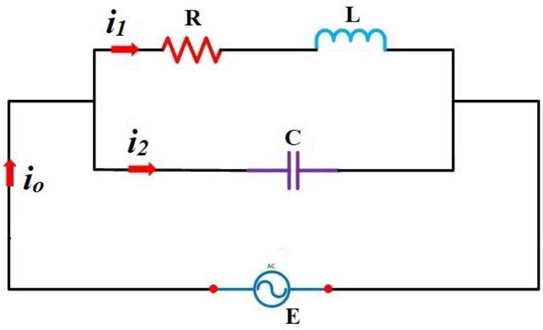
From Figure, $$\tan\phi = \frac{OB}{OA} = \frac{\left(E_L - E_C\right)}{E_R} = \frac{i_o\left(\omega L - \frac{1}{\omega C}\right)}{i_o R} = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$$

$$\phi = \tan^{-1}\left\{\frac{\omega L - \frac{1}{\omega C}}{R}\right\}$$

Here $\phi$ will lead or lag depending upon the values of $\omega L$ and $1/\omega C$.

**LCR PARALLEL RESONANT CIRCUIT**

The Parallel resonant circuit is as shown in figure. An inductance L and a resistance R are connected in series in one branch and a condenser C in another branch. A source of alternating e.m.f. is connected in the circuit. The current from the generator is $i_o$. From Kirchhoff's law




$i_0 = i_1 + i_2$

Let Z be the total impedance of the circuit.

Impedance of inductance and resistance branch, $Z_1 = (R + j\omega L)$

Impedance of condenser branch, $Z_2 = \frac{1}{j\omega C}$

The above two branches are in parallel. Hence, the resultant impedance is given by

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$\frac{1}{Z} = \frac{1}{(R + j\omega L)} + \frac{1}{1/j\omega C}$$

Admittance $Y = \frac{1}{Z} = \frac{1}{(R + j\omega L)} + \frac{1}{1/j\omega C} = \frac{(R - j\omega L)}{(R + j\omega L)(R - j\omega L)} + j\omega C$

$$\Rightarrow \text{Y}= \frac{(R - j\omega L)}{(R^2 + \omega^2 L^2)} + j\omega C = \frac{R}{(R^2 + \omega^2 L^2)} - \frac{j\omega L}{(R^2 + \omega^2 L^2)} + j\omega C$$

$$\Rightarrow \text{Y}= \frac{R}{(R^2 + \omega^2 L^2)} + j\left\{\omega C - \frac{\omega L}{(R^2 + \omega^2 L^2)}\right\}$$

The magnitude of admittance = $|\text{Y}| = \left[\left\{\frac{R}{(R^2 + \omega^2 L^2)}\right\}^2 + \left\{\omega C - \frac{\omega L}{(R^2 + \omega^2 L^2)}\right\}^2\right]^{1/2}$

From the above eq, Admittance is minimum (or impedance is maximum) when $\omega C = \frac{\omega L}{(R^2 + \omega^2 L^2)}$ or

$$C = \frac{L}{(R^2 + \omega^2 L^2)}$$

$$\Rightarrow CR^2 + C\omega^2 L^2 = L$$

$$\Rightarrow C\omega^2 L^2 = L - CR^2$$

$$\Rightarrow \omega^2 = L / L^2 C - CR^2 / CL^2$$

$$\Rightarrow \omega^2 = \frac{1}{LC} - \frac{R^2}{L^2} \Rightarrow \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

We know that $\omega = 2\pi f_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

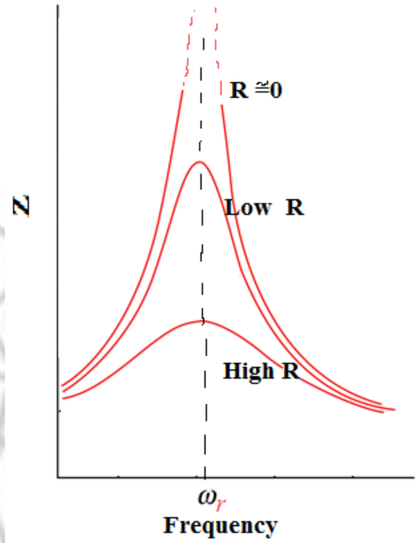
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

At this frequency, the admittance is minimum (impedance is maximum) and hence, the current is minimum. Such a frequency is called as resonant frequency. The circuit is known as parallel resonant circuit. At resonant frequency, the value of admittance of the circuit will be

$$|\mathrm{Y}_r| = \frac{R}{\left(R^2 + \omega^2 L^2\right)} \quad \text{or } \mathrm{Z}_r = \frac{\left(R^2 + \omega^2 L^2\right)}{R}$$

Substituting the value of $\omega^2$ we get

$$\mathrm{Z}_r = \frac{\left(R^2 + \left\{\frac{1}{LC} - \frac{R^2}{L^2}\right\}L^2\right)}{R} = \frac{L}{RC} = \text{Dynamic impedance}$$




This depends upon the value of resistance R. The smaller the value of R, the larger will be impedance. As R →0, the impedance Z→∞, The variation of impedance with frequency as shown in fig.

The circuit is called as a ***rejector circuit*** because it rejects only one frequency and excepts other.

**Quality Factor:**

Quality factor is defined as $2\pi$ times the ratio of energy stored in the system to average to average energy lost per period.

Energy stored in the circuit due to inductance is $\frac{1}{2}\mathrm{Li}^2$

Energy stored in the circuit due to capacitance is $\frac{1}{2}\mathrm{CE}^2$

Q- factor $Q = 2\pi \frac{\text{Energy stored in the system}}{\text{Energy lost per period}}$

$$Q = 2\pi \frac{\text{Energy stored in the system}}{\text{Power lost per sec x T}}$$

$$Q = \frac{2\pi}{\mathrm{T}} \frac{\text{Energy stored in the system}}{\text{Power lost per sec}}$$

$$= 2\pi f \frac{\frac{1}{2}\mathrm{Li}^2}{\frac{1}{2}\mathrm{i}^2 R}$$

$$= 2\pi f \frac{\frac{1}{2}\mathrm{Li}^2}{\frac{1}{2}\mathrm{i}^2 R} \qquad (\because P = Ei = iRi = i^2R)$$

$$= 2\pi f \frac{\mathrm{L}}{R}$$

$$Q = \frac{\omega \mathrm{L}}{R} = = \frac{\mathrm{X_L}}{\mathrm{R}}$$

$$= 2\pi f \frac{\frac{1}{2}\mathrm{Li}^2}{\frac{1}{2}\mathrm{i}^2 R} \qquad (\because P = Ei = iRi = i^2R)$$

$$= 2\pi f \frac{\mathrm{L}}{R}$$

$$Q = \frac{\omega \mathrm{L}}{R} = = \frac{\mathrm{X_L}}{\mathrm{R}}$$

Similarly, it can be shown that Q =1/$\omega$CR.

So, the quality factor may also be defined as the ratio of reactance of either inductance or capacitance to the resistance at the resonant frequency to the circuit. Since at resonance $X_L = X_C$, i.e, $\omega$L =1/$\omega$C, hence the value of Q is the same from both the expressions.

**Difference between series and parallel:**

| **Series Resonant Circuit** | **Parallel Resonant Circuit** |
|---|---|
| 1. In ideal case current is infinity | 1. In ideal case current is zero |
| 2. Series resonant frequency is given by $f_o = \frac{1}{2\pi\sqrt{LC}}$ | 2. Parallel resonant frequency is given by $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ |
| 3. At resonance, the power factor is unity and impedance is purely resistive $Z_r = R$ | 3. Power factor is also unity but the impedance is given by $Z_r$ =L/C R |
| 4. At resonant frequency current is maximum | 4. At resonant frequency current is minimum |
| 5. At resonant frequency impedance is minimum | 5. At resonant frequency impedance is maximum |
| 6. It is called accepter circuit | 6. It is called rejecter circuit |
| 7. At resonance, the circuit exhibits a voltage magnification and it is equal to Q-factor | 7. At resonance, the circuit exhibits a current magnification and it is equal to Q-factor |

**Power in AC circuit:**

Consider an AC circuit containing resistor, inductor and capacitor. Let the instantaneous values of voltage and current in the circuit are $E = E_0 \sin\omega t$ and $i = i_0 \sin(\omega t - \phi)$ respectively.

Instantaneous power = E × i

$$= E_0 \sin\omega t \; i_0 \sin(\omega t - \phi)$$

Average power consumed over one complete cycle = $\dfrac{\int_0^T E\, i\, dt}{\int_0^T dt}$

$$= \frac{\int_0^T E_0 \sin\omega t \; i_0 \sin(\omega t - \phi)\, dt}{\int_0^T dt} = \frac{E_0 i_0}{2T}\int_0^T 2\sin\omega t \; \sin(\omega t - \phi)\, dt = \frac{E_0 i_0}{2T}\int_0^T \left[\cos\phi - \cos(2\omega t - \phi)\right] dt$$

$$= \frac{E_0 i_0}{2T}\left[\cos\phi . t - \sin\left(\frac{2\omega t - \phi}{2\omega}\right)\right]_o^T = \frac{E_0 i_0}{2T}\left[\cos\phi . t - \sin\left(\frac{2.(2\pi/T)t - \phi}{2(2\pi/T)}\right)\right]_o^T$$

$$= \frac{E_0 i_0}{2T}\left[\cos\phi . T - \frac{\sin\left(2\left(\frac{2\pi}{T}\right)T - \phi\right)}{2\omega}\right] - \left[\cos\phi(0) - \frac{\sin\left(2\omega(0) - \phi\right)}{2\omega}\right]$$

$$= \frac{E_0 i_0}{2T}\left[T\cos\phi + \frac{\sin\phi}{2\omega}\right] - \left[0 + \frac{\sin\phi}{2\omega}\right]$$

$$= \frac{E_0 i_0}{2T}\left[T\cos\phi + \frac{\sin\phi}{2\omega} - \frac{\sin\phi}{2\omega}\right] = \frac{E_0 i_0}{2}\cos\phi = \frac{E_0}{\sqrt{2}} . \frac{i_0}{\sqrt{2}}\cos\phi$$

$$= E_{rms} . i_{rms} . \cos\phi$$

True power = Apparent power × power factor

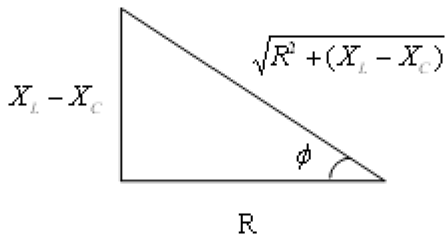
In this case cos ϕ is called power factor of the circuit for this circuit $\tan\phi = \dfrac{X_L - X_C}{R}$

$$\cos\phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

***Case i:*** If the circuit contains resistance only then ϕ is Zero.

Then power = $E_{rms}\, i_{rms} \cos 0$

Power = $E_{rms}\, i_{rms}$




***<u>Case ii:</u>*** If the circuit contains inductor only then $\phi = \pi/2$

Power = $E_{rms}\, i_{rms} \cos \pi/2 = 0$

If the circuit contains capacitor only then $\phi = \pi/2$

Power = $E_{rms}\, i_{rms} \cos \pi/2 = 0$

Hence if the circuit contains inductance or capacitance then the true power become zero and the current is called wattles current.

# UNIT-IV

# BASIC ELECTRONICS

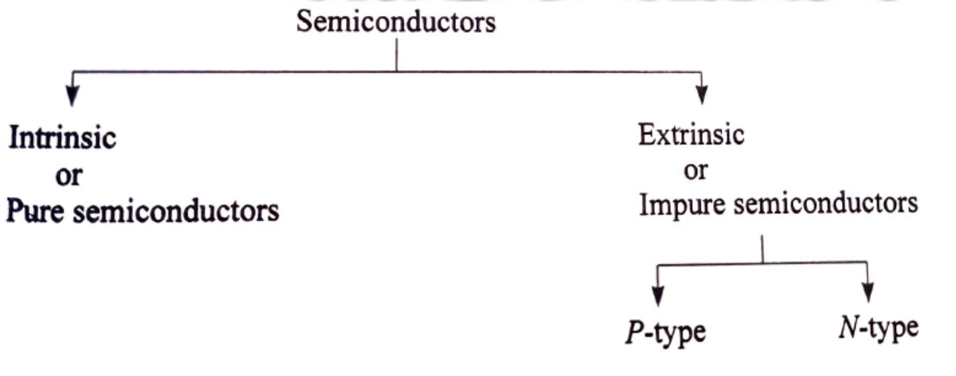
## SEMICONDUCTORS

A substance which has resistivity in between conductors and insulators is known as semiconductor.

For example germanium, silicon have resistivity ($10^{-5}$ to 0.5 ohm-meter) between good conductors like copper ($1.7 \times 10^{-8}$ ohm-metre) and insulators like glass ($9 \times 10^{11}$ ohm-metre). These substances are known as semiconductors. Some other peculiar properties also there that separate semiconductors from conductors, insulators and resistance materials.

These semiconductors may be classified as

(i) Pure or intrinsic semiconductors and (ii) Impure or extrinsic semiconductors




(1) Intrinsic Semiconductor:

These semiconductors are extremely in pure form. In these semiconductors electrons and holes are exclusively created by thermal excitation process. For example, pure crystals like germanium and silicon. For these semiconductors electron-hole pairs are formed while the electrons reaching the conduction band from valence band due to thermal excitation. So these semiconductors have equal number of electron-hole pairs.

(2) Extrinsic Semiconductor:

At room temperature, the intrinsic semiconductor has little current capability. In order to increase the conductivity, addition of some impurities are needed during the process of crystallization. Such semiconductor is called impurity or extrinsic semiconductor. The process of adding impurity to a semiconductor is known as doping.

Usually, the doping material is either pentavalent atoms [bismuth (83), antimony (51), arsenic (33), and phosphorus (15) which have 5 valence electrons] or trivalent atoms [gallium (31), indium (49), aluminum (13), and boron (5) which have 3 valence electrons].

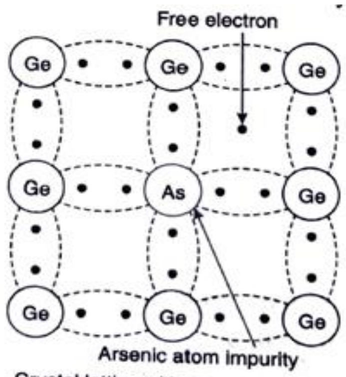
The pentavalent doping atom is known as donor atom because it donates one electron to the conduction band of pure semiconductor. The trivalent atom is, on the other hand, called as acceptor atom because it accepts one electron from semiconductor atom.

Depending on the type of impurity added, the extrinsic semiconductors can be divided into two classes.

(i) N-type semiconductor, and (ii) P-type semiconductor.

## (i). N-TYPE EXTRINSIC SEMICONDUCTOR

When a small amount of pentavalent impurity is added to a pure semiconductor crystal during the crystal growth, the resulting crystal is called as N-type extrinsic semiconductor. For example, a pentavalent arsenic is added to pure germanium crystal. Now the arsenic atom fits in the germanium crystal in such a way that its four valence electrons form a covalent bond with the four germanium atoms. The fifth electron of arsenic atom is not covalently bonded but it is loosely bound to the parent arsenic atom. This electron is available as a carrier of current

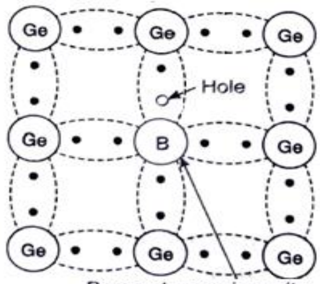


Crystal lattice with one germanium atom displaced by arsenic atom.

## (ii) P-TYPE EXTRINSIC SEMICONDUCTOR

When a small amount of trivalent impurity is added to a pure crystal during the crystal growth, the resulting crystal is called a P-type extrinsic semiconductor.

For example, when trivalent boron ($1s^2\ 2s^2 2p^1$) is added to pure germanium crystal. Each atom of boron fits into the germanium crystal with only three covalent bonds. In the fourth covalent bond, only germanium atom contributes one valence electron and there is deficiency of one electron which is called a hole. In other words, we can say that the fourth bond is incomplete due to shortage of one electron. Therefore, for each boron atom added, one hole is created.



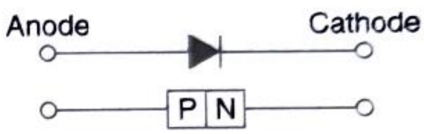
Crystal lattice with one germanium atom displaced by trivalent impurity atom boron

A small amount of boron provides millions of holes which constitute current.

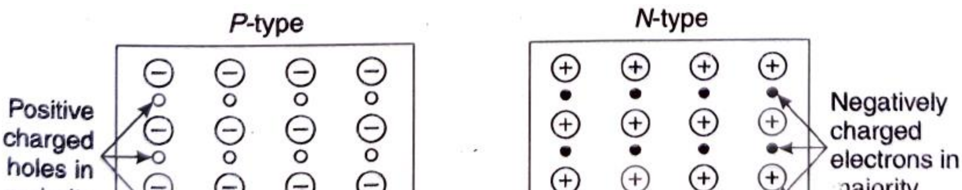
## **P-N JUNCTION DIODE**

This is a two terminal device made of Ge or Si crystal consisting of a P-N junction. A P-N junction is formed from a piece of Semiconductor (say germanium) by diffusing P-type material to one half side and N-type material to other half side. The plane dividing the two zones is known as a junction.

The P-type and N-type regions are referred to as anode and cathode respectively. When the diode is forward biased, the arrow head shows the conventional direction of current flow, i.e., the direction in which holes flow take place.



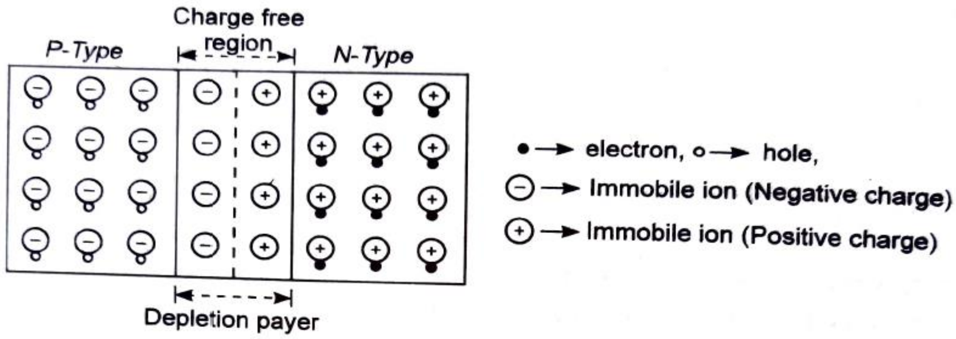

P-type and N-type semiconductor pieces before and after they are joined are shown in figure (a) and (b). P-type semiconductor has negative acceptor ions and positively charged free holes which move about on P-side. Similarly N-type semiconductor has positive donar ions and negatively charged free electrons which move about N-side. The Junction is a border where P-type and N-type regions meet.



(a)

## Depletion Layer

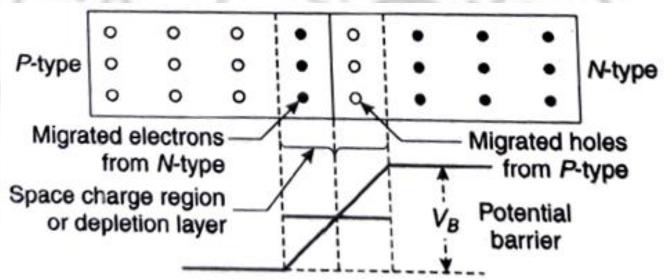
Now let us consider that the two pieces are joined together. As P-type material has a high concentration of holes and N-type material has high concentration of free electrons, hence there is a tendency of holes to diffuse over to N-side and electrons to P-side. The process is known as diffusion. So due to diffusion, some of the holes from P-side cross over to N-side where they combine with electrons




and become neutral and vice-versa.

In this region there is no charge available for conduction. So, charge carriers (electrons and holes) are depleted. It is termed as depletion layer or charged free region or space charge region.

## Potential Barrier




The diffusion of holes and electrons continues till a potential barrier is developed in charged free region. This prevents further diffusion or neutralization. The potential barrier can be increased or decreased by applying an external voltage.

**Junction Voltage**

When the depletion layer is formed, there are negative immobile ions in P-type semiconductor and positive immobile ions in N-type semiconductors. Now due to charge separation, a voltage $V_B$ is developed across the junction under equilibrium condition. This voltage is known as junction voltage or internal voltage. For Germanium it is generally 0.1 to 0.3 V.

## WORKING OF DIODE

The working of a diode is happened in two connections.

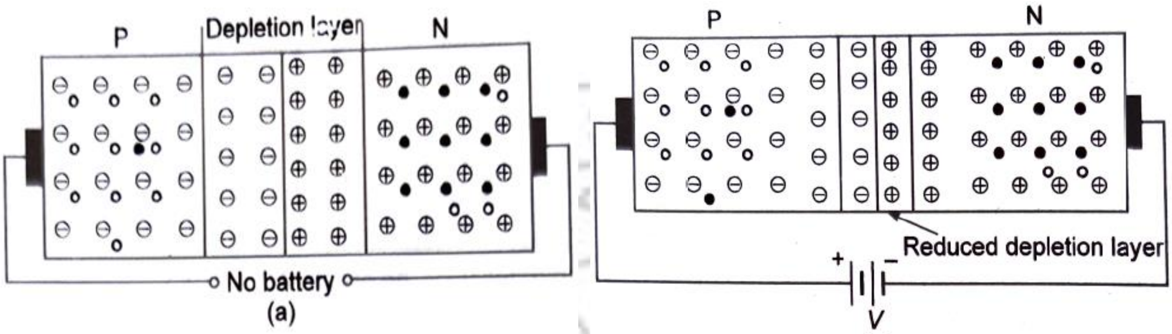
(1) Forward Bias and (2) Reverse Bias.

**(1) Forward Bias**

When an external voltage is applied to P-N junction in such a direction that it cancels the potential barrier and permits the current flow is called as forward bias. To apply a forward bias, the positive terminal of a battery is connected to P-type while the negative terminal is connected to N-type of the diode.

The applied forward potential establishes an electric field opposite to the potential barrier. Therefore, the potential barrier is reduced as show in figure. As this is very small (0.3 volt for Ge and 0.7 V for Si) therefore, a small forward voltage is sufficient to completely eliminate the barrier. When potential barrier is eliminated by the forward voltage, junction resistance becomes almost zero.

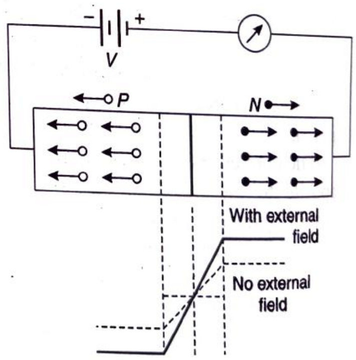
In case of the forward bias, the holes from P-type semiconductor are repelled by the positive battery terminal towards the junction and simultaneously, the electrons in N-type semiconductor are repelled by negative battery terminal towards junction. If the battery voltage is more than junction voltage, these carriers overcome the potential barrier and cross the junction to combine. When an electron-hole combination takes place near the junction, a covalent bond near the positive terminal of battery breaks down. This causes the discharge of an electron which enters the positive terminal. This action creates a



(a) (b)

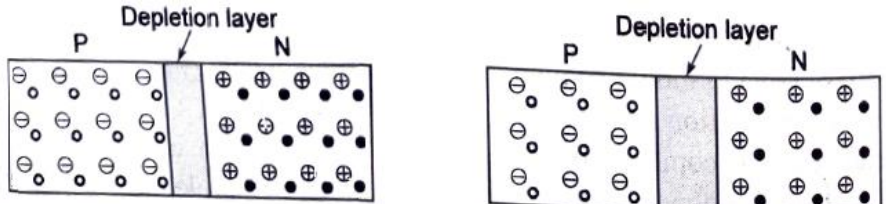
new hole which moves towards the junction. For each electron in N region that combines with a hole from the negative terminal of the battery. This constant movement of electrons towards the positive terminal and the holes towards the negative terminal produces a high forward current.

**Reverse Bias**




When an external voltage is applied to P-N junction in such a direction that it increases the potential barrier, then it is called as *reverse bias*. For reverse bias, the positive terminal of the battery is connected to N-type and negative terminal to P-type of the diode as shown in figure. The applied reverse voltage establishes an electric field which acts in the same direction of potential barrier. Therefore, the resultant field at the junction is strengthened and the potential barrier is increased. The potential barrier prevents the flow of charge carriers across the junction. In this way a high resistance path is established.

When the junction is reversed biased, the electrons in N-type semiconductor and holes in P-type semiconductor are attracted away from the junction. Since there is no recombination of electron hole pairs no current flows in the circuit.



(a) Depletion layer without reverse bias (b) Depletion layer with reverse bias

(i) When P-N junction is forward biased, it has a low resistance path and hence current flows in the circuit. On the other hand when it is reverse biased, it has high resistance path and no current flows in the circuit.

(ii) P-N junction diode is one way device which offers a low resistance when forward biased and behave like an insulator when reverse biased. So it can be used as a rectifier (converts A.C into D.C).

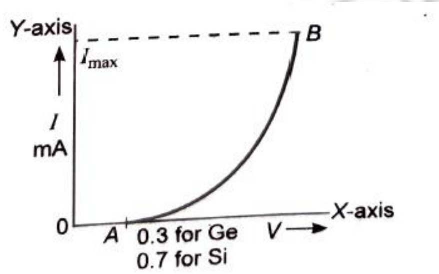
## VOLT-AMPERE CHARACTERISTICS OF P-N JUNCTION

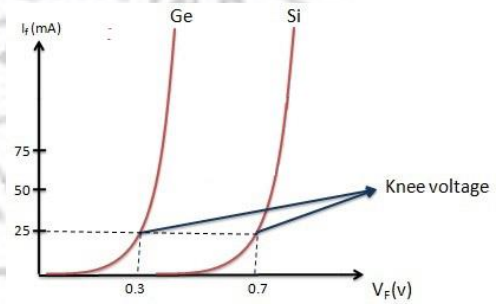
The characteristics are studied under the following two heads

(i) Forward bias, and (ii) Reverse bias.

### Forward Bias

For the forward bias of a P-N junction, P-type is connected to the positive terminal while the N-type is connected to the negative terminal of a battery as shown in figure. The potential at P-N junction can be varied with the help of potential divider. At some forward voltage (0.3 for Ge and 0.7 V for Si), the potential barrier is eliminated and current starts flowing. This voltage is known as threshold voltage ($V_{th}$) or cut-in voltage or knee voltage. It is practically same as barrier voltage $V_B$. For this the current flow is negligible.

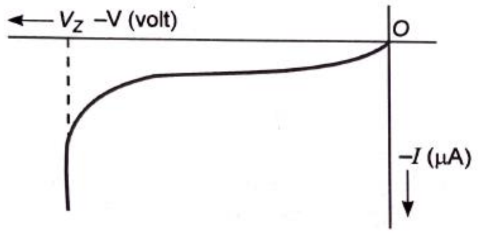



As the forward applied voltage increases beyond threshold voltage, the forward current rises exponentially. The current is recorded with the help of milliammeter (mA) connected in series with diode. A graph is drawn with voltage (in volts) applied on X-axis and the corresponding current (in milli-amperes) is taken on Y-axis. The graph is shown in figure.

It should be remembered that if the forward voltage is increased beyond a certain safe value, it produces an extremely large current which may destroy the junction due to overheating.

### Reverse bias




For the reverse bias of P-N junction, P type is connected to negative terminal while N- type is connected to positive terminal of the battery. The reverse potential at P-N junction can be varied with the help of potential divider. In this case the junction resistance become very high and practically no current flows through the circuit. However, in practice a small amount of current flows in the circuit due to minority carriers. This is called reverse current. As the reverse voltage is increased from zero, the rivers current quickly rises to its maximum or saturation value. This gives rise to a current called surface leakage current. This is independent of temperature but depends on reverse voltage. If the reverse voltage is further increased the kinetic energy of electrons due to minority carriers becomes so high that they knock out electrons from the semiconductor atoms. At this stage breakdown of junction occurs and there is no sudden rise of reverse current. Now the junction is destroyed permanently. This reverse current is measured with micro ammeter connected in the circuit.

**Conclusion**

1. The width of depletion region increases with the reverse voltage increases.

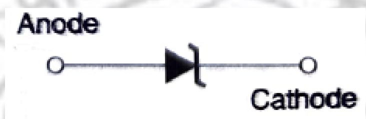
2. The junction offers a high resistance.

3. Practically no current flows in the circuit due to high resistance of the junction.

4. Small current flows through the junction due to minority charge carriers.

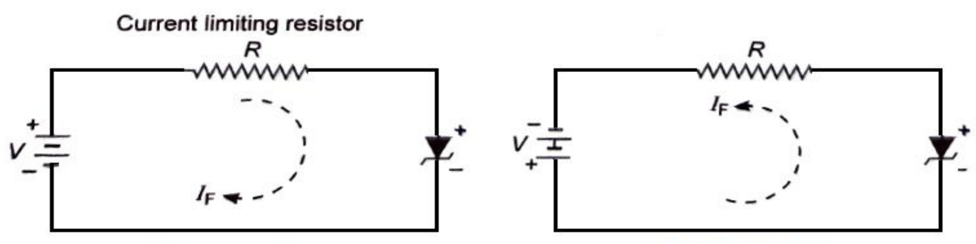
## ZENER DIODE

This is an example of junction device. Zener diode is a reverse biased heavily-doped silicon P-N junction diode which is operated in the breakdown region. Due to higher temperature and current capability, silicon is preferred in comparison to germanium. The symbol of a zener diode is shown in figure.




This is similar to a normal diode except the line representing the cathode is bent at both ends, i.e., like the letter Z for zener.

### Biasing of Zener Diode




(a) Forward bias (b) Reverse bias

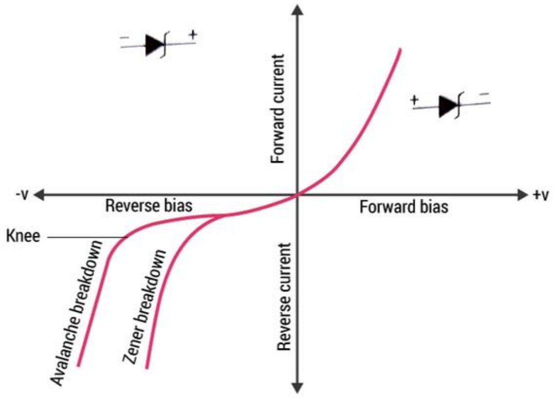
**(a) Forward biasing:**

For forward biasing, the anode is connected to positive terminal of battery while the cathode is connected to negative terminal of battery. This is identical to a forward biased diode while forward biasing. This biasing is generally not used for Zener.

**(b) Reverse biasing:**

In this case, negative terminal is connected to anode while the positive terminal is connected to cathode. Here, R is current limitation resistor. In reverse biasing, the zener diode is used as voltage regulator.

**V-I Characteristics of Zener Diode**




The V-I characteristic is shown in figure. As the reverse voltage applied to P-N junction is increased from zero, the current remains very small over a long range and increases very slightly with increasing bias voltage. But when the reverse bias is made very high, the covalent bonds near the junction breaks down. Now the reverse current increases abruptly to a large value. The corresponding voltage is called break down voltage or zener voltage. In this region of characteristic curve, the voltage across the diode remains constant over a large range of current. Hence, the zener diode may be used to stabilize voltage at a predetermined value.

**Difference between a Junction diode and a Zener diode**

To a junction diode, external voltage can be applied either under forward bias condition or under reverse bias condition while to a zener diode, reverse bias is only applied and operated in breakdown region.

There are two mechanisms of the breakdown.

(i) Zener breakdown

Zener breakdown takes place in very thin junction, i.e., when P-type and N-types are very heavily doped and consequently the depletion layer is narrow. When a small reverse bias voltage is applied, a very strong electric field (above $10^7$ V/m) is set up across the thin depletion layer. This field is enough to break the covalent bonds. Now extremely large number of electrons and holes are produced which constitute the reverse saturation current (zener current). This breakdown mechanism is called *zener breakdown*. Zener current is independent of the applied voltage and depends only on the external resistance.

(ii) Avalanche breakdown

This type of breakdown takes place when both sides of junction are lightly doped and consequently the depletion layer is large. In this case electric field across the depletion layer is not so strong to produce zener breakdown. Here minority charge carriers are accelerated by the field collision with the semiconductor atoms in the depletion region. So due to the collision with valence electrons, covalent bonds are broken and electron-hole pairs are generated. These new carriers so produced acquired energy from applied potential and in-turn produce additional carriers. This cumulative process is called as *avalanche multiplication* and the breakdown is called *avalanche breakdown*. This occurs at higher reverse voltages.

At reverse voltages less than 6 V, Zener breakdown predominates while at about 8 V, avalanche breakdown predominates. So a diode in Zener breakdown should be called as Zener diode whereas in avalanche breakdown it should be called as avalanche diode. When the breakdown voltage is reached in a zener diode, the current increases rapidly with additional voltage. As a result, a diode in breakdown maintains an almost constant voltage across itself over a wide range of current. Thus, the zener diode is most suited for voltage regulation.

**Applications**

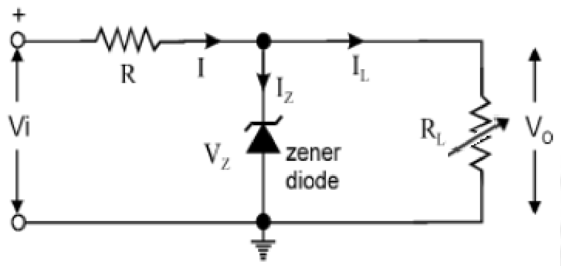
Following are few common applications of zener diode:

(i) Voltage regulation (ii) A peak clipper

(iii) Switching operation (iv) Meter protection

(v) As reference element

**ZENER DIODE AS VOLTAGE STABILIZER**




A simple circuit to stabilize voltage supplied to a load $R_L$ is as shown in figure. The zener diode is connected to a supply voltage through resistor R. The battery B reverse-biases the zener diode. The load $R_L$ is connected across the terminals of diode. The value of R is selected in such a way that in the absence of load $R_L$, maximum safe current flows in the diode as zener voltage.

As long as voltage across the load resistor $R_L$ is less than the breakdown voltage $V_z$, the zener diode does not conduct. When the voltage increases, the voltage drop across load $R_L$, (or zener diode) becomes greater than the Zener breakdown voltage. Now zener operates in breakdown region. The resistance R limits the current through Zener diode within the safe maximum limit $(I_z)_{max}$.

From Kirchhoff s law, $I = I_Z + I_L$ (1)

It is obvious from characteristics curve that voltage $V_Z$ across zener diode remains almost constant even when zener current varies considerably.

Let us consider that supply voltage $V_{in}$ increases. Now the current through zener diode and load resistance increases. At the same time, the zener diode resistance decreases and the current through it increases more than proportionately. As a result, a greater voltage drop occurs across R. As a consequence, the output voltage $V_{out}$ (or voltage across the diode) will become very close to the original value. The reverse is also true.

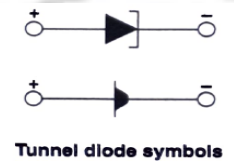
Thus, a zener diode maintains constant voltage across the load as long as the supply voltage is more than zener voltage.

Let us now consider the case when the load resistance $R_L$ decreases for constant input voltage $V_{in}$. As a result, load current $I_L$ increases. It is clear from eq. (1) that $I_Z$ will decrease by the same amount so that remains the same. This keeps the voltage drop across series resistance R constant. Therefore, output voltage remains constant. On the other hand, if load resistance $R_L$ increases, the load current $I_L$ decreases. Now, the Zener diode allows an extra current such that I remains the same. As a result, output voltage $V_{out}$ remains constant.

Thus, zener diode maintains a constant voltage across the load even whenever there is change in load resistance.

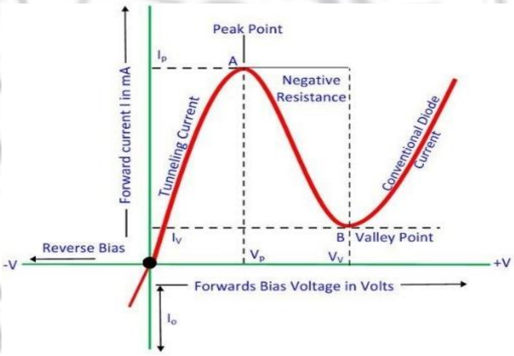
## TUNNEL DIODE

Esaki, a Japanese scientist in 1957 announced a new diode which utilizes the phenomenon of tunnelling and hence it is called as *tunnel diode* or *Esaki diode*. A conventional P-N diode is doped by impurity atoms in the concentration 1 part in $10^8$. If the concentration of impurity atoms is greatly increased in a P-N junction (by about 1000 times) then the depletion layer width reduces to about $10^{-6}$ cm and the device characteristics are completely changed. Under this condition, many carriers punch through the junction even when they do not have enough energy to overcome the potential barrier (0.3 V for Ge and 0.7 V for Si). Consequently, large forward current is produced even though the applied bias is much less than 0.3 V or 0.7 V. This phenomenon is known as tunnelling.




## Volt-Ampere Characteristics

Figure shows the V-I characteristic of tunnel diode. As soon as the forward bias is applied, significant current is produced. The current quickly reaches its peak value $V_p$. It is represented by point A. This is due




to quantum mechanical tunneling of electrons through a small narrow space region in the junction. So the voltage increases from 0 to $V_p$ and current increases from 0 to $I_p$.

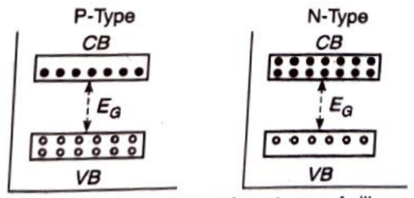
When forward voltage is further increased, the diode current starts decreasing. The current decreases to $I_v$ corresponding the valley voltage $V_v$. It is indicated with point B. Thus from point A to B, current decreases as voltage increases. This is the negative resistance region. Here instead of absorbing power, negative resistance produces power. This unique property make the diode useful in very high frequency oscillators. For the voltages above $V_v$, the current starts increasing as a conventional P-N junction diode.

If the tunnel diode is reversed biased then it acts like a good conductor, i.e., the reverse current increases with increasing reverse voltage.

So, current can be obtained in three different voltage regions. This feature makes the tunnel diode useful in pulse and digital circuits.

**Tunneling Theory:**

The tunnelling phenomenon can be explained by considering the energy band diagram of P-type and N-type semiconductor materials. In P-type material, due to heavy doping, there is an increased concentration of holes in valence band and similarly for N-type material there is an increased of electrons in conduction band. When P-type and N-type materials are joined, the energy level diagram is as shown in

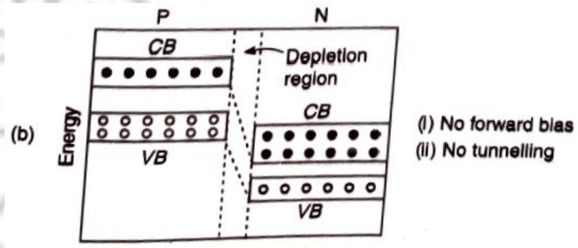


(a) Energy band diagram of two types of silicons

figure.

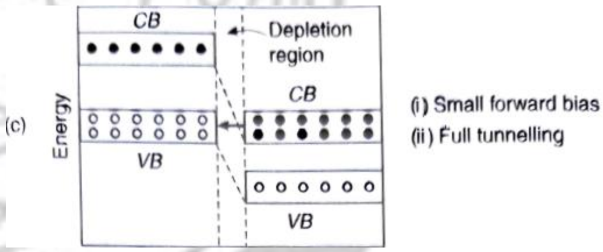
*1. No Forward Bias*

Till no forward bias is applied, there is a rough alignment of their respective valence and conduction bands. The energy levels of holes in P-region are slightly out of alignment with the energy levels of conduction electrons in N-region of the junction. No current flows across the junction.




*2. Small Forward Bias*

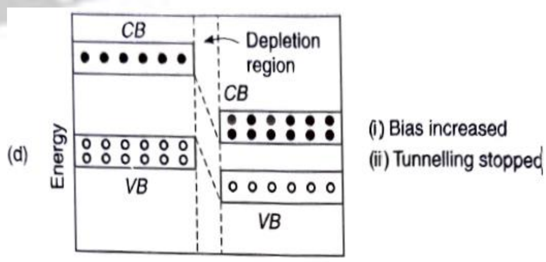
When a small forward voltage (= 0.1 V) is applied, the energy bands in N-region upwards relative to P-type. Now there is an exact alignment of bands on both side and a very small gap between conduction band of N-type and valance band of P-type as shown in figure. At this stage, electrons tunnel through the depletion layer with the velocity of light and give rise to large current.




This tunneling current reaches a maximum value $I_p$ at a forward bias $V_p$ of the order of 0.1 volt.

*3. Large Forward Bias*

After $V_p$, as the applied voltage is increased, the current starts decreasing because the energy level of N-region are raised so high that two bands are out of alignment. Now the distance between conduction band of N-type and valance band of P-type increases. So, only part of the electrons in the conduction band see energy level across the barrier, i.e., In this case tunneling is stopped. The current reaches the minimum value (called valley current). Between peak current $I_p$ and valley current $I_v$, a negative dynamic resistance is obtained. At bias voltage $V_v$, tunneling is completely stopped and if we rise the voltage above $V_v$, current increases with voltage and works as ordinary junction diode.




Tunneling is much faster than a normal crossing. This enables a tunnel diode to switch ON and OFF faster than an ordinary diode. Due to this reason, tunnel diode is extensively used in special applications where very fast switching speeds are required like in high speed computer memory, high frequency oscillators, etc.

Advantages

(i) This can withstand very large temperature changes.

(ii) It requires low power.

(iii) Small size and low cost.

(iv) It has long life and low noise.

(v) Speed is high.

(vi) It has environmental immunity.

Disadvantages

(i) There is no isolation between input and output. So, this leads to serious circuit design difficulties.

(ii) It is a two terminal device.

(iii) It has low output voltage swing.

Applications

The main applications are as follows:

(i) This is used as logic memory storage device.

(ii) It is an ultra-high speed switching device. This is possible due to tunneling mechanism, which occurs at the speed of light. The switching time is of the order of nano-second (= $10^{-9}$ sec).

(iii) This is used in relaxation oscillator due to its negative resistance.

(iv) This is used in microwave oscillator at frequencies of the order of 10 GHz. This is possible due to low values of inductance and capacitance.

| **Feature** | **Zener diode** | **Tunnel diode** |
|---|---|---|
| Construction | Made up of silicon with two layers (P-N) | Made up of germanium or gallium arsenide with two layers (P-N) |
| Doping | Heavily doped | Heavily doped |
| Operation | Acting as breakdown device at reverse bias | Exhibits negative resistance at forward bias |
| Applications | Constant voltage source, Voltage regulator | Microwave oscillator, Ultra high speed switching device |

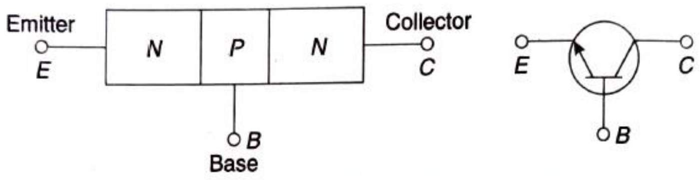
## THE BIPOLAR JUNCTION TRANSISTOR

A junction transistor is simply a sandwich of one type of semiconductor material between two layers of the other type. Accordingly, there are two types of transistors.

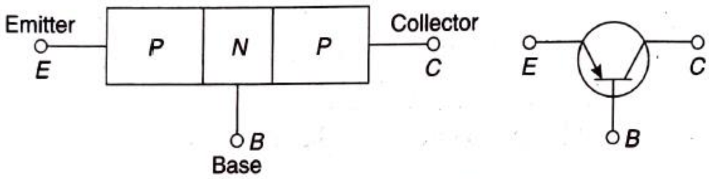
1. N-P-N transistor

2. P-N-P transistor.

When a layer of P-type material is sandwiched between two layers of N-type material, the transistor material is known as N-P-N transistor. Similarly, when a layer of N-type material is sandwiched between two layers of P-type material, the transistor is known as P-N-P transistor.



(a) Structure and symbol of *N-P-N* transistor



(b) Structure and symbol of *P-N-P* transistor

Transistors are made either from silicon or germanium crystal. The symbolic representation of N-P-N and P-N-P transistors are shown in figures.

Although the two outer regions are of the same type but they cannot be interchanged. The reason is that the two regions have different physical and electrical properties. The collector region is made physically larger than emitter region. The base is very thin and lightly doped. The emitter is heavily doped while the doping of collector is between the heavy doping of emitter and light doping of the base.

A transistor has the following sections:

**(i) Emitter:** This forms the left hand section or region of the transistor. The main function of this region is to supply majority charge carriers (either electrons or holes) to the base and hence it is heavily doped in comparison to other regions.

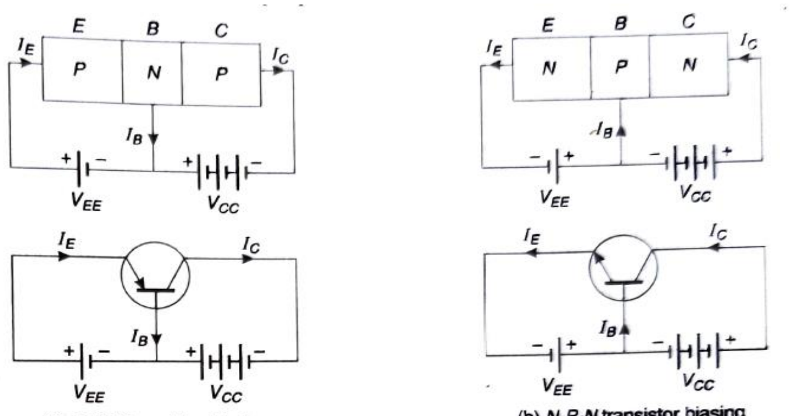
**(ii) Base:** The middle section of the transistor is known as base. This is very lightly doped and very thin ($10^{-6}$ m) as compared to either emitter or collector so that it may pass most of the injected charge carriers to the collector.

**(iii) Collector:** The right hand section of the transistor is called as collector. The main function of the collector is to collect majority charge carriers through the base. This is moderately doped. As regards the symbols, arrowhead is always at the emitter. The direction indicates the conventional direction of current flow, i.e., in case of N-P-N transistor it is from base to emitter (base is positive with respect to emitter) while in case of P-N-P transistor it is from emitter to base (emitter is positive with respect to base).

**TRANSISTOR BIASING**

In a circuit, transistor can be biased such a way that the emitter-base junction is always forward-biased while the collector-base junction is always reversed biased. For this purpose a battery $V_{EE}$ is connected between emitter and base while a battery $V_{CC}$ is connected between collector and base. The emitter-base junction of P-N-P is forward biased by transistor connecting the positive terminal of $V_{EE}$ to emitter and negative terminal to base. P-N-P transistor is reverse-biased by connecting the negative terminal of $V_{CC}$ to collector while positive terminal to base. Similarly, for N-P-N transistor, emitter-base junction is connected in forward bias by connecting positive terminal of $V_{EE}$ to base and negative terminal to emitter. For reverse bias, positive terminal of $V_{CC}$ connected to collector while negative terminal to base.

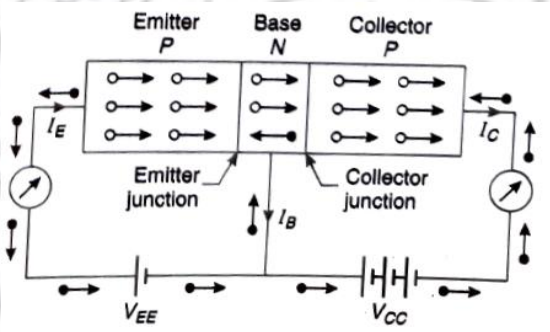
So, emitter circuit has low resistance while collector circuit has high resistance. In a transistor, it transfer a signal from low resistance to high resistance circuit.




(a) *P-N-P* transistor biasing (b) *N-P-N* transistor biasing

OPERATION OF P-N-P TRANSISTOR

In a PNP transistor, emitter-base junction is forward biased and collector-base junction as reverse biased. The Operation of PNP transistor is as follows:




The holes of P region (emitter) are repelled by positive terminal of battery $V_{EE}$ towards the base. A forward potential barrier at emitter junction is reduced as it is in forward biased and hence holes cross this junction and penetrates N-region. This creates the emitter current $I_E$. The width of the base region is very thin and it is lightly doped and hence only two to five per cent of the holes recombine with the free electrons of N-region. This constitutes the very small base current $I_B$. The remaining holes (95% to 98%) able to drift across the base and enter the collector region. They are swept up by the negative collector voltage $V_{CC}$. They constitute the collector current $I_C$.

As each hole reaches the collector electrode, an electron is emitted from the negative terminal of battery and neutralizes the hole. Now, a covalent bond near the emitter electrode breaks down. The liberated electron enters the positive terminal of battery $V_{EE}$ while the hole immediately moves towards the emitter junction. This process is repeated again and again. Here it should be remembered that:

(i) Current conduction within PNP transistor takes place by hole conduction from emitter to collector. i.e., majortiy charge carriers in a PNP transistor are holes. The conduction in the external circuit is carried out by electrons.

(ii) The collector current is slightly less than the emitter current. This is due to the fact that 2 to 5% of the holes are lost in recombination with electron in base region. Thus, the collector current is slightly less than emitter current.

(iii) The collector current is a function of emitter current, i.e., with the increase or decrease in the emitter current, a corresponding change in collector current is observed.

Besides the hole current, there is electron current which flows from base region to emitter region. This current depends upon emitter base potential. As the width of the base region is very small, the ratio of hole current to electron current is very small. So for all practical purposes, the electron current may be neglected.

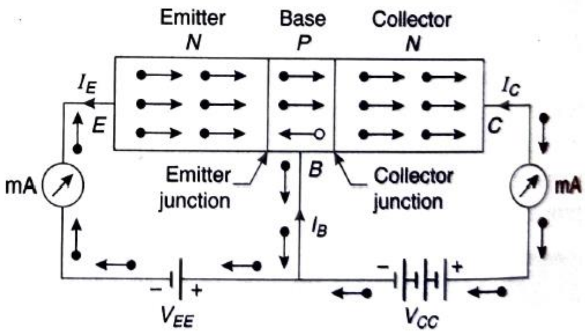
Thus, only the hole current plays an important role in the operation of PNP transistor.

Therefore,

$I_E = I_B + I_C$

## OPERATION OF N-P-N TRANSISTOR

The biasing of a NPN transistor is shown in Figure. The emitter junction is forward biased because electrons are repelled from the negative emitter battery terminal $V_{EE}$ towards the junction. The collector junction is reverse biased because electrons are flowing away the collector junction towards the positive collector battery terminal $V_{CC}$.




The electron in the emitter region are repelled from the negative terminal of battery towards the emitter junction. Since the potential barrier at the junction is reduced due to forward bias and base region is very thin and lightly doped, electrons cross the P-type base region. A few electrons combine with the holes in P-region and are lost as charge carriers. Now the electrons in N region (collector region) readily swept up by the positive collector voltage $V_{CC}$.

For every electron flowing out the collector and entering the positive terminal of battery $V_{CC}$ electron from the negative emitter battery terminal enters the emitter region. In this way electron conduction takes place continuously so long as the two junctions are properly biased.

So, the current conduction in NPN transistor is carried out by electrons.

Therefore,

$I_E = I_B + I_C$

Here, according to sign convention,

(i) the emitter current $I_E$ is positive while base current $I_B$ and collector current $I_C$ both are negative in PNP transistor.

$I_E - I_C - I_B = 0$

$I_E = I_C + I_B$

(ii) In case of NPN transistor, emitter current $I_E$ is negative while the base current $I_B$ and collector current $I_C$ both are positive.

$-I_E + I_C + I_B = 0$

$I_E = I_B + I_C$

**TRANSISTOR CIRCUIT CONFIGURATIONS**

Following are the three types of transistor circuit configurations

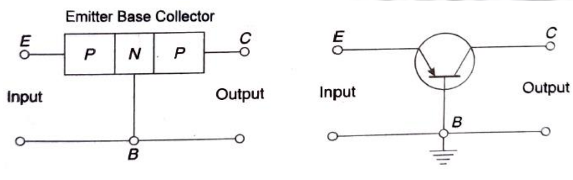
(1) Common-base (CB)

(2) Common-emitter (CE)

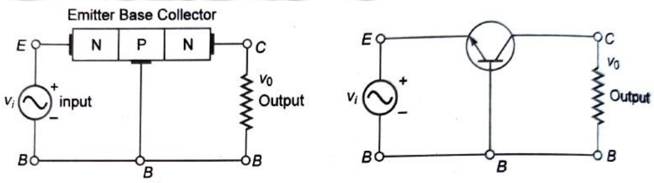
(3) Common-collector (CC)

Here the term Common is used to denote the transistor lead which is common to the input and output circuits. This is because when a transistor is connected in a circuit, four terminals are required (two for input and two for output) while a transistor has only three terminals. The difficulty is removed by making one terminal of the transistor 'common' to both input and output terminals. The common terminal is generally grounded. Each configuration has specific advantages and disadvantages. In any circuit configuration, the emitter is always forward-biased while the collector is always reverse-biased.

**COMMON-BASE (CB) CONFIGURATION**

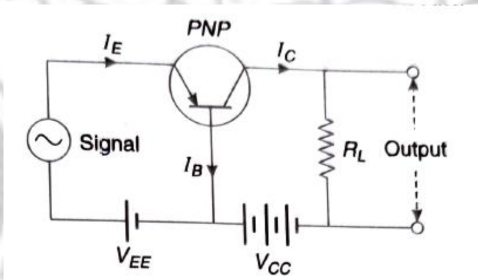



CB Configutation for PNP Transistor




CB Configutation for NPN Transistor

In this configuration the input signal is applied between emitter and base while the output is taken from collector and base. As base is common to input and output circuits, hence the name common base configuration.




**Current amplification factor ($\alpha$)**

(i) When no signal is applied

The ratio of the collector current to the emitter current is called d.c. alpha ($\boldsymbol{\alpha_{dc}}$) of a transistor

$$\alpha_{dc} = -\frac{I_C}{I_E}$$

(negative sign signifies that $I_E$ flows into transistor while $I_C$ flows out of it)

If we write $\alpha_{dc}$ simply by $\alpha$, then

$$\alpha = -\frac{I_C}{I_E} \tag{1}$$

(ii) When signal is applied

The ratio of change in collector current to the change in emitter current at constant collector base voltage is defined as current amplification factor

$$\alpha_{ac} = \frac{\Delta I_C}{\Delta I_E}$$

$\alpha$ of a transistor is a measure of the quality of a transistor. The higher the value of $\alpha$, the better is the transistor in the sense that collector current approaches the emitter current. For all physical purposes, $\alpha_{dc} = \alpha_{ac} = \alpha$ and practical values in commercial transistors range from 0.9 to 0.99.

From eq. (1), considering only magnitudes of the currents

$$I_C = \alpha . I_E$$

$$I_B = I_E - I_C$$

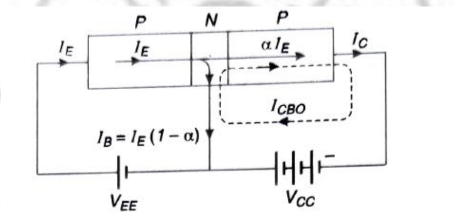
$$I_B = I_E - \alpha\, I_E = I_E\,(1-\alpha) \qquad \ldots(2)$$

**Total collector current**

The collector current consists of the following two parts:

(i) The current produced by normal transistor action, i.e., component controlled by emitter current, this is due to the majority carrier and its value is $\alpha I_E$.

(ii) The leakage current $I_{leakage}$. This current is due to the motion of minority carrier across base-collector junction due to reverse bias. This is much smaller than $\alpha I_E$. The leakage current is abbreviated as $I_{CBO}$ i.e.,

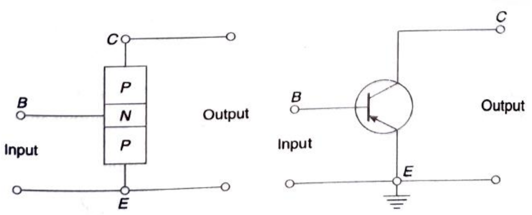



collector base current with emitter open.

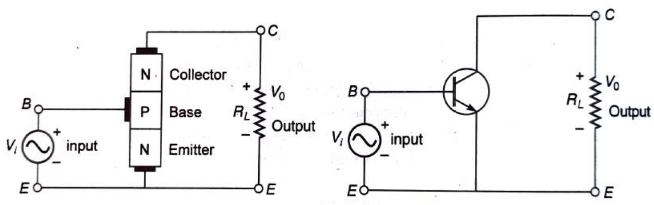
$$\therefore \textbf{Total collector current} = \underset{Majority}{\boldsymbol{\alpha I_E}} + \underset{Minority}{\boldsymbol{I_{CBO}}}$$

If $I_E = 0$, (emitter circuit is open), even there will be small leakage current $I_{CBO}$ in the collector circuit. The current $I_{CBO}$ is usually small and may be neglected in transistor circuit calculations.

**COMMON-EMITTER (CE) CONFIGURATION**




CE configuration for PNP transistor

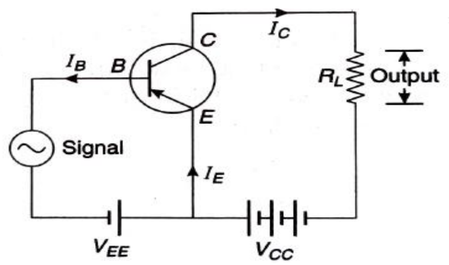



CE configuration for NPN transistor

In this configuration, the input signal is applied between base and emitter and the output is taken from collector and emitter. As emitter is common to input and output circuits, hence the name common emitter configuration.

Current Amplification Factor ($\beta$)

(i) When no signal is applied




The ratio of collector current to the base current called dc beta ($\beta_{dc}$) of transistor

$$\beta_{dc} = \beta = \frac{I_C}{I_B} \quad (1)$$

(ii) When signal is applied

The ratio of change in collector current to the change in base current is defined as base current amplification factor. Thus,

$$\beta_{dc} = \beta = \frac{\Delta I_C}{\Delta I_B}$$

From eq. (1), $$I_C = \beta I_B \quad (2)$$

For almost all transistors, the base current is less than 5% of the emitter current. Due to this fact, $\beta$ is generally greater than 20. $\beta$ ranges from 20 to 500. Hence, this configuration is frequently used for the circuits where current gain as well as voltage gain is required.

**Total Collector Current**

Total collector current $$I_C = \beta I_B + I_{CEO} \quad (3)$$

where $I_{CEO}$ is the leakage current. Even though $I_B = 0$ there is a leakage current from collector- emitter junction. It is called $I_{CEO}$, the subscript CEO stands for collector to emitter with base open.

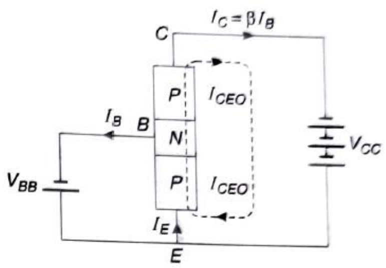
We know that

$I_E = I_B + I_C$ and $I_C = \alpha I_E + I_{CBO}$

$$I_C = \alpha (I_B + I_C) + I_{CBO}$$

$$I_C (1-\alpha) = \alpha I_B + I_{CBO}$$

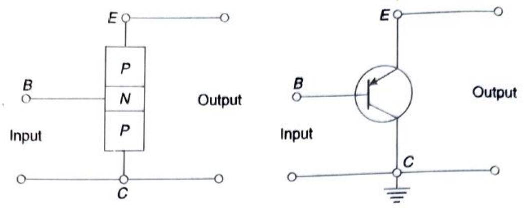
$$I_C = \frac{\alpha}{1-\alpha} I_B + \frac{1}{1-\alpha} I_{CBO} \quad \textbf{(4)}$$



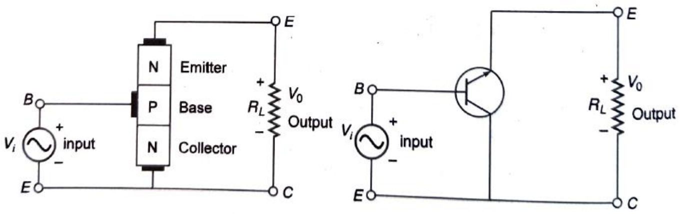

From, eq. (3) and (4), we have

$$\beta = \frac{\alpha}{1-\alpha} \text{ and } I_{CEO} = \frac{1}{1-\alpha}$$

## COMMON-COLLECTOR (CC) CONFIGURATION




**CC configuration for PNP transistor**

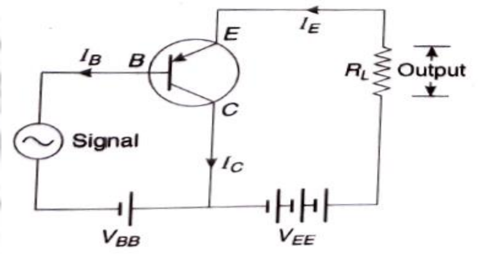



**CC configuration for NPN transistor**

In this configuration, the input signal is applied between base and collector and the output is taken from emitter. As collector is common to input and output circuits, hence the name common collector configuration.

**Current amplification factor ($\gamma$)**

(i) When no signal is applied




The ratio of emitter current to the base current is called as dc. gamma ($\boldsymbol{\gamma_{dc}}$) of the transistor.

$$\gamma_{dc} = \gamma = \frac{I_E}{I_B}$$

(ii) When signal is applied

The ratio of change in emitter current to the change in base current is known as current amplification factor $\gamma$

$$\gamma = \frac{\Delta I_E}{\Delta I_B}$$

This configuration provides about the same current gain as common emitter circuit as $\Delta I_E \approx \Delta I_C$ but the voltage gain is always less than one.

**Total Emitter Current**

We know that

$$I_E = I_B + I_C$$
$$I_C = \alpha\, I_E + I_{CBO}$$
$$I_E = I_B + (\alpha\, I_E + I_{CBO})$$
$$= I_B + \alpha\, I_E + I_{CBO}$$
$$I_E\,(1-\alpha) = I_B + I_{CBO}$$
$$I_E = \frac{I_B}{(1-\alpha)} + \frac{I_{CBO}}{(1-\alpha)}$$
$$I_E = (1+\beta)\, I_B + (1+\beta)\, I_{CBO}$$
$$\frac{1}{(1-\alpha)} = (1+\beta).$$

**Application**

This configuration has very high input resistance (750 $K\Omega$) and very low output resistance (25Ω). So the voltage gain is always less than one. Hence, this configuration is seldom used for amplification. The most important use is for impedance matching, i.e., for driving a low impedance load from a high impedance source.

**RELATIONS BETWEEN $\alpha$, $\beta$ and $\gamma$**

1. Relation between $\alpha$ and $\beta$.

We know that

$$\alpha = \frac{I_C}{I_E} \text{ and } \beta = \frac{I_C}{I_B}$$

$$I_E = I_B + I_C \text{ or } I_B = I_E - I_C$$

$$\beta = \frac{I_C}{I_E - I_C} = \frac{I_C/I_E}{1-(I_C/I_E)}$$

$$\beta = \frac{\alpha}{1-\alpha} \qquad (1)$$

Cross-multiplying eq. (1), we get

$$\beta(1-\alpha) = \alpha$$

$$\text{or } \beta - \beta\alpha = \alpha$$

$$\beta = \alpha(1+\beta)$$

$$\alpha = \frac{\beta}{1+\beta} \qquad (2)$$

It can be seen that $1-\alpha = \frac{1}{1+\beta}$ (3)

2. Relation between $\gamma$ and $\alpha$

We know that

$\gamma = \frac{I_E}{I_B}$ and $\alpha = \frac{I_C}{I_E}$

$$I_B = I_E - I_C$$

$$\gamma = \frac{I_E}{I_E - I_C} = \frac{1}{1-(I_C/I_E)}$$

$\gamma = \frac{1}{1-\alpha}$ (4)

3. Relation between $\gamma$ and $\beta$

According to eq. (3), we have

$$(1-\alpha) = \frac{1}{(1+\beta)}$$

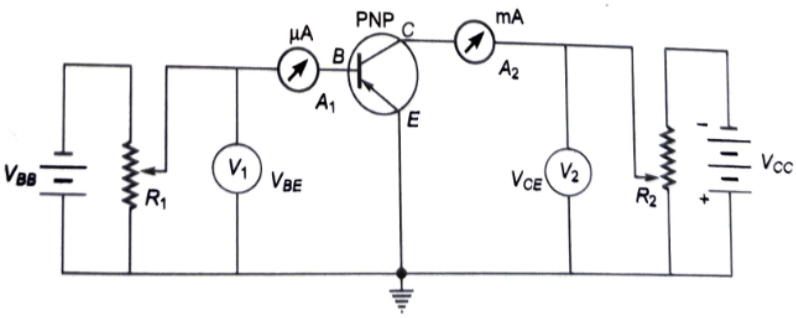
Substituting this value in eq. (4), we have

$$\gamma = \frac{1}{1-\alpha} = (1+\beta) \qquad (5)$$

**Conclusion:**

$$\beta = \frac{\alpha}{1-\alpha}, \alpha = \frac{\beta}{1+\beta}, \gamma = \frac{1}{1-\alpha}, \gamma = (1+\beta)$$

## CHARACTERISTICS OF COMMON EMITTER TRANSISTOR CIRCUIT




In the circuit, the battery $V_{BB}$ provides forward bias to emitter-base junction with the help of potential divider $R_1$. The voltmeter $V_1$ measures the base-emitter voltage $V_{BE}$. The microammeter $A_1$ measures the base current $I_B$.

A Battery $V_{CC}$ is connected between collector and emitter through a potential divider $R_2$. The positive terminal of the battery is connected to emitter while the negative terminal is connected to the collector so that the collector is reverse-biased. The voltmeter $V_2$ measures the collector-emitter voltage $V_{CE}$ and the milliammeter $A_2$ measures the collector current.

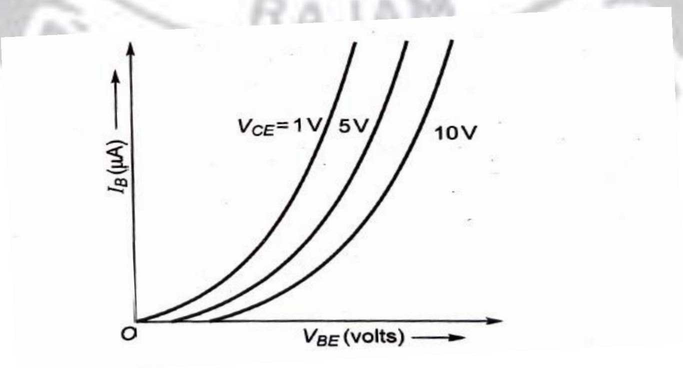
We consider the following characteristics:

(1) Input characteristics (2) Output characteristics

### (1) Input Characteristics

The curve between base current ($I_B$) and base-emitter voltage ($V_{BE}$) at constant collector-emitter voltage ($V_{CE}$) represents the input characteristic. For plotting the input characteristic, the collector-emitter voltage $V_{CE}$ is fixed at particular value. The base emitter voltage $V_{BE}$ is varied with the help of potential divider $R_1$ and the base current $I_B$ is noted for each value of $V_{BE}$.

A graph of $I_B$ against $V_{BE}$ is drawn. The curve so obtained is known as input characteristic. The experiment is repeated for other fixed values of $V_{CE}$.




Important points from the characteristic:

(i) The characteristic be similar to that of a forward-biased diode curve. This is because of the base-emitter section (like a junction diode) of transistor connected in forward-bias.

(ii) In this case, $I_B$ increases less rapidly with $V_{BE}$ as compared to common-base configuration. This shows that the input resistance of common-emitter circuit is higher than that of common-base circuit.

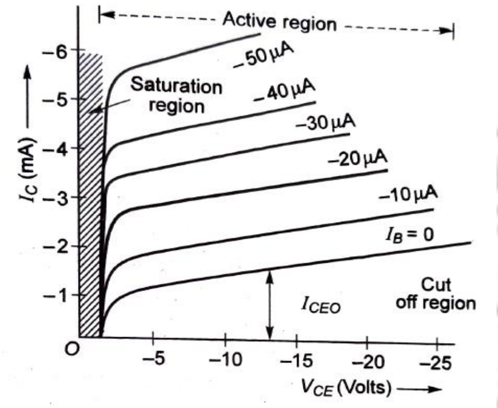
Input resistance:

The ratio of change in base emitter voltage ($V_{BE}$) to the resulting change base current ($I_B$) at constant collector-emitter voltage ($V_{CE}$) is defined as input resistance. This denoted by $r_i$. Therefore,

$$r_i = \left.\frac{\Delta V_{BE}}{\Delta I_B}\right|_{V_{CE}=\text{constant}}$$

**(2) Output Characteristics**

The curve between collector current $I_C$ and collector emitter voltage $V_{CE}$ at constant base current $I_B$ represents the output characteristic.




For plotting output characteristic, the base current $I_B$ is kept fixed. With the help of potential divider $R_2$, the value of $V_{CE}$ is varied in steps and the collector current $I_C$ is noted for each value of $V_{CE}$. A graph of $I_C$ against $V_{CE}$ is drawn. The curve obtained is known as output characteristic. The experiment is repeated for different values of $I_B$.

**Important points:**

(i) In the active region (collector junction is reverse-biased while emitter junction is forward-biased), for small values of base current, the effect of collector voltage over collector current is small while for large base current values this effect increases. Thus, the current gain of this configuration is greater than unity. In this configuration, the transistor is used as an amplifying device, but it must be restricted in the active region only.

(ii) When $V_{CE}$ has very low value (ideally zero), the transistor is said to be saturated and it operates in the saturation region of characteristic. In the saturated region, the change in base current $I_B$ does produce a corresponding change in collector current $I_C$.

(iii) When $V_{CE}$ is increased too far, collector-base junction completely breaks down and due avalanche breakdown, collector current increases rapidly. In this case, the transistor is damaged.

(iv) In the cutoff region, a small amount of collector current flows even when base current $I_B$ =0, is called $I_{CEO}$. Since, main collector current is zero, the transistor is said to be cutoff.

Output resistance

The ratio of change in collector-emitter voltage ($V_{CE}$) to the resulting change in collector current $I_C$ at constant base current is defined as output resistance. This is denoted by $r_0$.

$$r_0 = \left.\frac{\Delta V_{CE}}{\Delta I_C}\right|_{I_B=\text{constant}}$$

**HYBRID PARAMETERS OR h-PARAMETERS**

Hybrid means "'mixed". These parameters have mixed dimensions and hence they are called as hybrid parameters. These parameters are very useful at high frequencies. At high frequencies transistor has low input impedance and high output impedance. Here Y and Z parameters becomes difficult. In this case, $i_1$ and $v_2$ taken as independent variables while $v_1$ and $i_2$ are taken as dependent variables. Hence,

$$v_1 = f(i_1, v_2) \qquad (1)$$

$$i_2 = f(i_1, v_2) \qquad (2)$$

Taking the total differentials, we get

$$dv_1 = \frac{\partial v_1}{\partial i_1} di_1 + \frac{\partial v_1}{\partial v_2} dv_2 \qquad (3)$$

$$di_2 = \frac{\partial i_2}{\partial i_1} di_1 + \frac{\partial i_2}{\partial v_2} dv_2 \qquad (4)$$

For small a.c. signals limited to the quasi-linear region the partial derivatives become constants. Thus,

$$V_1 = h_{11}\, I_1 + h_{12}\, V_2 \qquad (5)$$

$$I_2 = h_{21}\, I_1 + h_{22}\, V_2 \qquad (6)$$

In matrix form

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \qquad (7)$$

Appropriate choice of open circuit ( $I_1 = 0$) and short circuit ($V_2 = 0$) conditions applied to eqs. (5) & (6).

If we short circuit the output terminals, i.e., $V_2 = 0$, now eqs. (5) & (6) become

$$V_1 = h_{11}\, I_1 + h_{12}\,(0) \Rightarrow h_{11} = \frac{V_1}{I_1}$$

$$I_2 = h_{21}\, I_1 + h_{22}\,(0) \Rightarrow h_{21} = \frac{I_2}{I_1}$$

1f we Open circuit the input terminals, i.e., $I_1 = 0$, then

$$V_1 = h_{11}(0) + h_{12}\, V_2 \Rightarrow h_{12} = \frac{V_1}{V_2}$$

$$I_2 = h_{21}(0) + h_{22}\, V_2 \Rightarrow h_{22} = \frac{I_2}{V_2}$$

So h-parameters are defined as

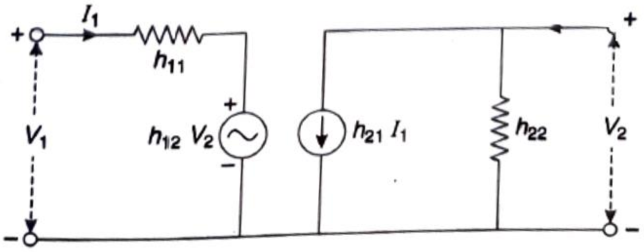
$$h_{11} = h_i = \left.\frac{\partial v_1}{\partial i_1}\right|_{v_2=0} = \frac{V_1}{I_1} = \text{Input Impedance with output short circuited}, V_2 = 0$$

$$h_{12} = h_r = \left.\frac{\partial v_1}{\partial v_2}\right|_{i_1=0} = \frac{V_1}{V_2} = \text{Reverse voltage ratio with input open circuited},\ I_1 = 0$$

$$h_{21} = h_f = \left.\frac{\partial i_2}{\partial i_1}\right|_{v_2=0} = \frac{I_2}{I_1} = \text{Forward current gain with output short circuited}, V_2 = 0$$

$$h_{22} = h_o = \left.\frac{\partial i_2}{\partial v_2}\right|_{i_1=0} = \frac{I_2}{V_2} = \text{Output admittance with input open circuited},\ I_1 = 0$$

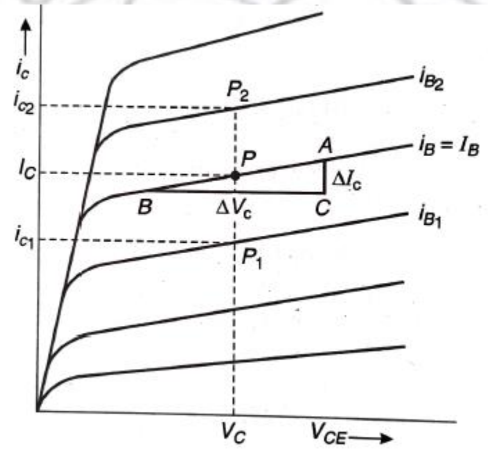
**Equivalent Circuit**




The equivalent circuit derived on the basis of eqs. (5) and (6) is shown in figure. The input circuit resistance $h_{11}$ in series with a voltage generator $h_{12}$ $V_2$. The output circuit consists of a current $h_{21}I_1$ and shunt resistance $h_{22}$. The input portion is a Thevenin equivalent (voltage with series resistance while the output portion is a Norton equivalent (current generator with shunt resistance). Thus, the circuit is mixed, i.e. hybrid. In common emitter configuration, these h-parameters are represented with $h_{ie}$, $h_{re}$, $h_{fe}$ and $h_{oe}$.

**<u>DETERMINATION OF h-PARAMETERS FROM STATIC CHARACTERISTICS</u>**

<u>DETERMINATION OF PARAMETERS $h_{fe}$ AND $h_{oe}$</u>




The output characteristic curves of a transistor in common emitter configuration is as shown in figure. The curve between collector current $I_C$ and collector emitter voltage $V_{CE}$ at constant base current represents the output characteristic. For plotting output characteristic, the base current $I_B$ is kept fixed. With the help of potential divider, $V_{CE}$ is varied in steps and the corresponding collector current $I_C$ is noted for each value of $V_{CE}$. A graph of $I_C$ against $V_{CE}$ is drawn.

As shown in the figure, different curves are drawn by changing the value of $I_B$. In this figure, we consider the curve for $i_B = I_B$. At a point P on this curve, the collector current and collector voltage are $I_C$ and $V_C$ respectively. Through the point P we draw a vertical straight line which cuts the curves for $i_{B1}$, and $i_{B2}$, and at points $P_1$ and $P_2$ respectively. The corresponding collector currents are $i_{C1}$ and $i_{C2}$, respectively

Using the definitions of $h_{fe}$ and $h_{oe}$, we have

$$h_{fe} = \frac{\partial i_C}{\partial i_B} = \left.\frac{\Delta i_C}{\Delta i_B}\right|_{V_C} = \frac{i_{C_2} - i_{C_1}}{i_{B_2} - i_{B_1}} \quad (1)$$

$$h_{oe} = \frac{\partial i_C}{\partial v_C} = \left.\frac{\Delta I_C}{\Delta v_C}\right|_{I_B} = \frac{AC}{BC} \quad (2)$$

DETERMINATION OF PARAMETERS $h_{re}$ AND $h_{ie}$

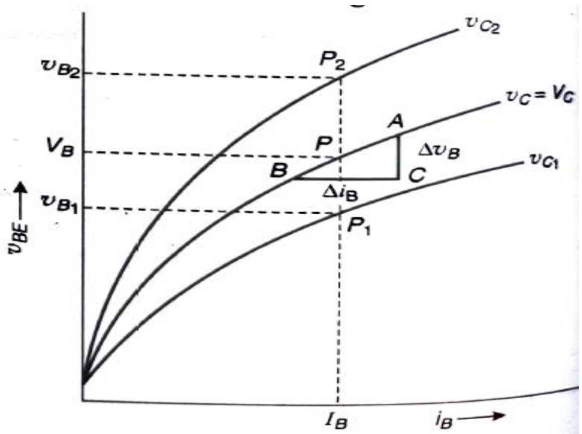


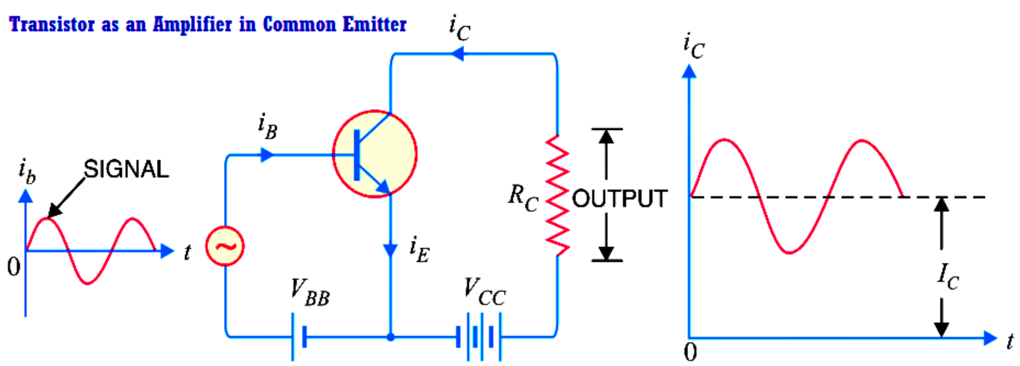

Figure shows the input characteristics of a transistor in common-emitter configuration. The curve between base current $I_B$ and base-emitter voltage $V_{BE}$ at constant collector-emitter voltage $V_{CE}$ represents the input characteristic. For plotting the input characteristic, the collector-emitter voltage $V_{CE}$ is kept fixed. A graph is drawn between $I_B$ and $V_{BE}$ by changing the value of $V_{CE}$ with the help of potential divider. From figure, we consider the curve for $v_C = V_C$. At a point on this curve, the quiescent base voltage and base current are $V_B$ and $I_B$. We draw a vertical line through this point P which cuts the curves for $v_{C_1}$ and $v_{C_2}$ at points $P_1$ and $P_2$ respectively. The corresponding values of base voltages are $v_{B_1}$ and $v_{B_2}$, respectively.

$$h_{re} = \frac{\partial v_B}{\partial v_C} = \left.\frac{\Delta v_B}{\Delta v_C}\right|_{i_B} = \frac{v_{B_2} - v_{B_1}}{v_{C_2} - v_{C_1}} \qquad (3)$$

$$h_{ie} = \frac{\partial v_B}{\partial i_B} = \left.\frac{\Delta v_B}{\Delta i_B}\right|_{V_C} = \frac{AC}{BC} \qquad (4)$$

Hence, the slope of the appropriate input characteristic at the quiescent point P gives $h_{ie}$. This can be evaluated by drawing a straight line tangential to the input characteristics at point P. This may also be calculated by drawing incremental triangle ABC about point P and noting the values of AC and BC.

## TRANSISTOR AS AN AMPLIFIER




The transistor amplifier circuit is as shown in figure. Here, the weak signal to be amplified is applied between base-emitter circuit and the output is taken across the load resistor $R_L$, connected in the collector circuit. To achieve faithful amplification, the input circuit is always in forward bias. For this purpose, base bias battery $V_{BB}$ is connected to set input circuit always forward biased. A small change in signal voltage produces appreciable change in the base current which causes the same change in collector current. When the collector current flows through the load resistance $R_L$, a large voltage is developed across it. In this way a weak signal is applied in the input circuit and amplified signal is obtained as output.

Let a small voltage change $\Delta V_i$ between base and emitter causes a relatively large emitter current change $\Delta I_E$. The current amplification factor is given by

$$\beta = \frac{\Delta I_C}{\Delta I_B}$$

$$\Delta I_C = \beta \, \Delta I_B$$

The change in output voltage across the load resistor

$$\Delta V_o = R_L \times \Delta I_C$$

$$= R_L \times \beta \, \Delta I_B$$

Under these circumstances, the voltage amplification

$$A = \frac{\Delta V_o}{\Delta V_i}$$

If the dynamic resistance of the emitter junction is $r_e$, then $\Delta V_i = r_e \times \Delta I_B$

$$\therefore A = \frac{\Delta V_o}{\Delta V_i} = \frac{R_L \times \beta \, \Delta I_B}{r_e \times \Delta I_B} = \frac{\beta . R_L}{r_e}$$

**Voltage gain ($A_V$):**

The voltage gain is defined as the ratio of change in output voltage ($\Delta V_{CE}$) to the change in input voltage ($\Delta V_{BE}$) when transistor is connected in common emitter configuration.

$$A_V = \frac{\Delta V_{CE}}{\Delta V_{BE}} = \frac{Change\ in\ output\ current\ \times effective\ load}{Change\ in\ input\ current\ \times input\ resistance}$$

$$= \frac{\delta I_C \times R_L}{\delta I_B \times R_i}$$

$$= \beta \left(\frac{R_L}{R_i}\right)$$

**Power gain ($A_P$):**

This is defined as the ratio of output signal power to input signal power.

$$A_P = \frac{(\delta I_C)^2 \times R_L}{(\delta I_B)^2 \times R_i}$$

$$= \left(\frac{\delta I_C}{\delta I_B}\right)\left(\frac{\delta I_C \times R_L}{\delta I_B \times R_i}\right)$$

$$= \beta \times A_V$$

So, Power gain = current gain × voltage gain

# UNIT-5

# DIGITAL ELECTRONICS

## 5.1. Digital Electronics

The digital electronics is a branch of electronics which deals with the generation, processing and storage of digital signals. This helps in the analysis, design and construction of digital systems. Any variation that can be transmitted through space is called a signal. Basically there are two types of signals i.e., analogue signal and digital signal. Analogue signals are those continuously changing from minimum value to maximum values. On the other hand, there are another type of signals which are confined to a limited number of discrete levels of current or voltage. These signals are called as digital signals. These signals are coded as 0 or 1 numbers which correspond to OFF (low voltage) or ON (high voltage) conditions. These are provided in a switch. George Boole is the inventor of digital electronics. The algebra which is used in digital electronics is called as Boolean algebra. In digital electronics, any information can be represented in terms of zero's and one's. It is represented by a bit.

In Analog circuits, the transistors as used as amplifiers of sinusoidal signal. For every change in the input circuit, the linear amplifier produces a corresponding change in output circuit. So, the circuit acts as a linear circuit. In Digital circuits, transistor can be used as a switching circuit, i.e., it can be used to operate as ON and OFF switch. The magnitude and waveform of output have no relation with those of input. So, the circuit acts as a non-linear circuit.

## 5.2. NUMBER SYSTEM -SOME BASIC TERMS

A number system is a code. Each distinct quantity is assigned to a symbol. After memorizing the code, we can perform arithmetic operations. Hence, a number system gives relations between quantities and symbols.

(i) Digit: It is the basic symbol used in a number system.

(ii) Base: It is the number of digits or basic symbols in a number system. The decimal system has a base of ten while the binary number system has a base of two.

(iii) Bit: It is an abbreviated form of Binary digit. $E_g$: 1010 has four binary digits or four bits.

1 nibble = 4 bits, l byte = 8 bits

(iv) Radix: It is a synonym of word base.

(v) Weight: It refers the position of a digit in a number. Each position has different decimal value in the number system.

### 5.2.1 TYPES OF NUMBER SYSTEMS

The common numbers systems used in digital electronics are as follows.

(1) Decimal system (0 to 9)

(2) Binary system (0 or 1)

(3) Octal system (0 to 7)

(4) Hexa-decimal system (0 to 9, A, B, C, D, E, F)

**Non suitability of decimal system in digital circuit:**

Decimal system is not suitable for carrying out in digital circuits. This is because the supply voltages using semiconductor devices is of the order of few volts. Hence, ten levels will naturally become crowded. Any adjoining voltage levels will be quite close to each other resulting in level to be mixed. This will lead to misinterpretation of voltage level. Whereas in case of binary system there will be only two fixed voltage levels: One is close to ground level (say bit 0) and the other is close to supply voltage $V_{CC}$ (say bit 1). These two levels are fairly apart from each other and hence, there is no possibility of misinterpretation of data. Generally, in positive logic, 0 to 1 volt represents the lower level or 0 and 3 to 5 volt represents the higher level (i.e. 1).

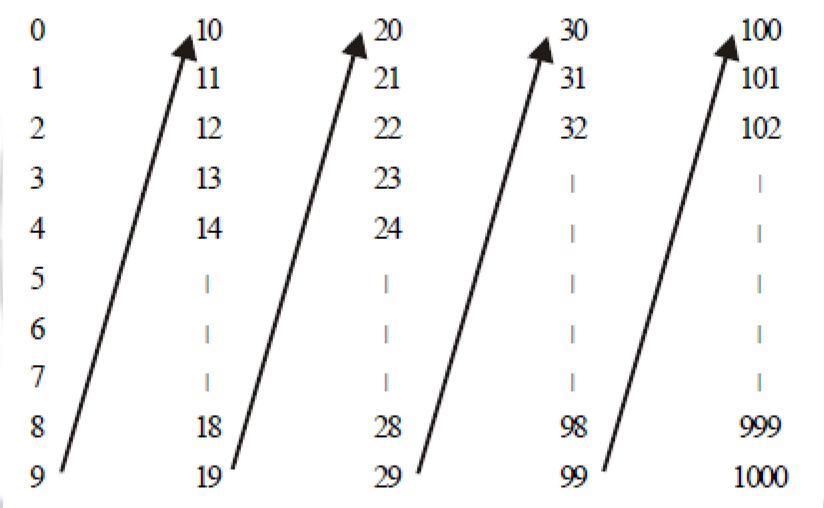
### 5.2.2. THE DECIMAL NUMBER SYSTEM

This is the most commonly used number system in our daily life developed by Aryans around 5th century. It contains 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, i.e., it has a base or radix 10.

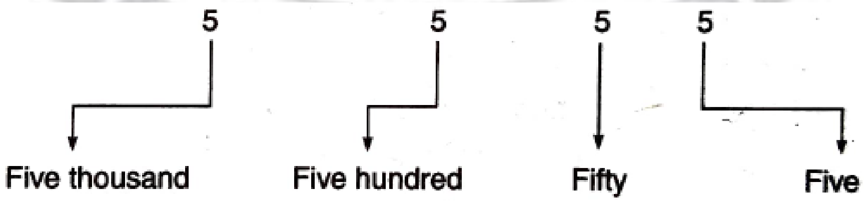
Even though the absolute value of each digit is fixed, its decimal value depends on its position value or place value or weight.

In decimal system, we have single digit numbers from 0 to 9. To get double digit number, after 9, we use the second decimal digit, i.e., 1 followed by the first digit, i.e., 0 resulting in 10.

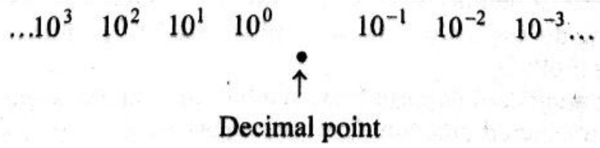
To get 11, 12, 13, ..., etc., we use second digit followed by the second, third, fourth , etc. This process goes up to 19. After this we use the third decimal digit (i.e., 2) followed by first, second, third,..etc. to get 20, 21, 22, 23,..., etc. In this way we can go up to 99. Similar logic follows for 100 or above.




*Counting in a decimal number system.*




The weights or place values of different digits in a mixed decimal numbers are as follows




$1989 = 1 \times 10^3 + 9 \times 10^2 + 8 \times 10^1 + 9 \times 10^0$

↓ ↓ ↓ ↓

1000 100 10 1

(Weights of digits in positions)

Here number 9 is least significant digit (LSD) and 1 is the most significant digit (MSD).

In general, a decimal number system can be expressed as

$$N = a_n\ 10^n + a_{n-1}\ 10^{n-1} + a_{n-2}\ 10^{n-2} + .. + a_1\ 10^1 + a_0\ 10^0 + a_{-1}\ 10^{-1} + ... + a_{-m}\ 10^{-m}$$

$$N = \sum_{-m}^{n} a_i\ 10^i$$

For example

(coefficients)

$$2021.0825 = 2 \times 10^3 + 0 \times 10^2 + 2 \times 10^1 + 1 \times 10^0 + 0 \times 10^{-1} + 8 \times 10^{-1} + 2 \times 10^{-2} + 5 \times 10^{-3}$$

(weights)

**5.2.3. BINARY NUMBER SYSTEM**

In binary system, there are only two digits, i.e., 0 and 1. To express a number, the same decimal procedure is adopted here. Thus, to express 2, we use the second binary digit (1) followed by the first (0). This gives 10 (read as one zero and not ten).

Similarly, 3 can be represented by 11 (one-one). Now the two binary digits are exhausted. Thus, to get 4 in the binary system, we use second digit followed by two first digits, i.e., 100 (one-zero-zero). Similarly, 5 can be expressed as 101 (one-zero-one) and so on.

| Decimal | Binary |
|---|---|
| 0 | 0000 |
| 1 | 0001 |
| 2 | 0010 |
| 3 | 0011 |
| 4 | 0100 |
| 5 | 0101 |
| 6 | 0110 |
| 7 | 0111 |
| 8 | 1000 |
| 9 | 1001 |

Binary number 11111000101 can be written as

$$11111000101 = 1 \times 2^{10} + 1 \times 2^9 + 1 \times 2^8 + 1 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

The binary weights in a mixed binary number may be listed below

$$\dots\quad 2^3\quad 2^2\quad 2^1\quad 2^0\quad \bullet\quad 2^{-1}\quad 2^{-2}\quad 2^{-3}\quad \dots$$

Binary point

Any Binary number can be represented as

$$N = a_n\ 2^n + a_{n-1}\ 2^{n-1} + a_{n-2}\ 2^{n-2} + .. + a_1\ 2^1 + a_0\ 2^0 + a_{-1}\ 2^{-1} + ... + a_{-m}\ 2^{-m}$$

$$N = \sum_{-m}^{n} a_i\ 2^i$$

Here $a_i$ values are either 0 or 1.

### 5.2.4. THE OCTAL NUMBER SYSTEM

The word 'octal' means eight. So, in the octal number system, the base or radix is 8, i.e., eight distinct counting digits 0, 1, 2, 3, 4, 5, 6, 7 are used to express all possible numbers. These digits 0 to 7 have exactly the same physical meaning as in decimal system. The highest digit in octal system is seven (7).

For counting beyond 7, a two-digit combinations are formed. The combinations are formed by taking second digit followed by first, then by second and so on. Thus after 7, the next octal number will be 10 (second digit followed by first), then 11 (second digit followed by second) and so on.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 |

$$\leftarrow 8^3 \; 8^2 \; 8^1 \; 8^0 \,\bullet\, 8^{-1} \; 8^{-2} \; 8^{-3} \rightarrow$$

$\uparrow$

Octal point

The position value (or weight) for each digit is given by different powers of 8 as shown below

Any number in octal system can be represented as

$$N = a_n\, 8^n + a_{n-1}\, 8^{n-1} + a_{n-2}\, 8^{n-2} + .. + a_1\, 8^1 + a_0\, 8^0 + a_{-1}\, 8^{-1} + ... + a_{-m}\, 8^{-m}$$

$$N = \sum_{-m}^{n} a_i\, 8^i$$

For example, decimal equivalent of 258 is

| | | |
|---|---|---|
| 2 | 5 | 8 |
| $8^2$ | $8^1$ | $8^0$ |
| 64 | 8 | 1 |

$$(258)_8 = 2\times 64 + 5\times 8 + 8\times 1 = (176)_{10}$$

### 5.2.5 THE HEXADECIMAL NUMBER SYSTEM

The word hexadecimal is composed of hexa meaning 6 and decimal meaning 10. Hence, the word Hexadecimal means sixteen. Thus, its base or radix is 16. The maximum value of any digit in this system is 15, i.e., 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.

Here A stands for 10, B stands for 11, C stands for 12. D stands for 13, E stands for 14, and F stands for 15. After reaching F, we take the second digit followed by first digit, then second followed by second, then second followed by third and so on.

Any number in hexa decimal system can be represented as

$$N = a_n\, 16^n + a_{n-1}\, 16^{n-1} + a_{n-2}\, 16^{n-2} + .. + a_1\, 16^1 + a_0\, 16^0 + a_{-1}\, 16^{-1} + ... + a_{-m}\, 16^{-m}$$

$$N = \sum_{-m}^{n} a_i\, 16^i$$

| Hexadecimal | Decimal | Hexadecimal | Decimal |
|---|---|---|---|
| 0 | 0 | 10 | 16 |
| 1 | 1 | 11 | 17 |
| 2 | 2 | 12 | 18 |
| 3 | 3 | 13 | 19 |
| 4 | 4 | 14 | 20 |
| 5 | 5 | 15 | 21 |
| 6 | 6 | 16 | 22 |
| 7 | 7 | 17 | 23 |
| 8 | 8 | 18 | 24 |
| 9 | 9 | 19 | 25 |
| A | 10 | 1A | 26 |
| B | 11 | 1B | 27 |
| C | 12 | 1C | 28 |
| D | 13 | 1D | 29 |
| E | 14 | 1E | 30 |
| F | 15 | 1F | 31 |

## 5.3. DECIMAL TO BINARY CONVERSION

### 5.3.1. Converting Integer Numbers from Decimal-to-Binary

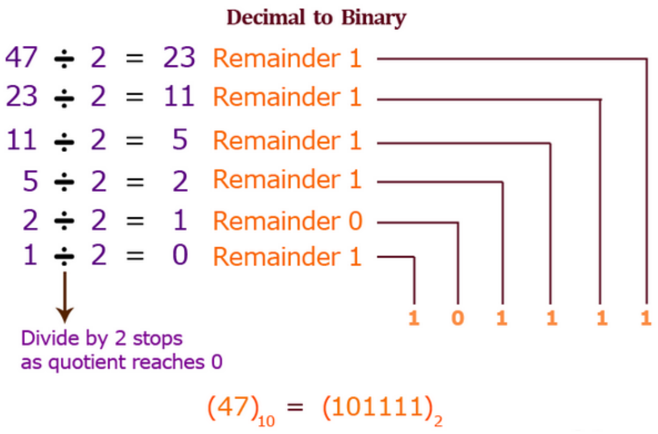
It is a systematic way of converting integer numbers from decimal-to-binary. The procedure for conversion is as given below:

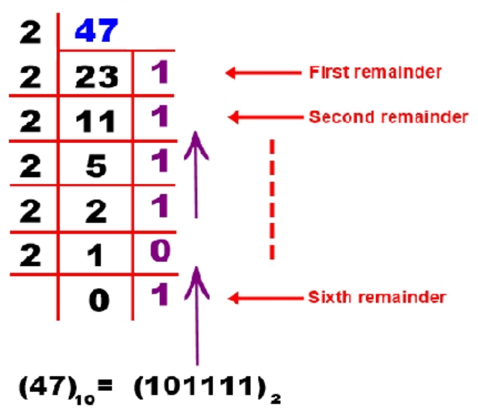
Step 1. Begin by dividing the given decimal number by 2.

Step 2. Divide each resulting quotient by 2 until there is a 0 whole number quotient.

Step 3. The remainders generated by each division form the binary number. The first remainder to be produced is the least significant bit (LSB) in the binary number, and the last remainder to be produced is the most significant bit (MSB). In other words, reading the remainders from bottom-to-top constitutes the required binary number.

Example 1: Conversion of decimal 47 to its equivalent binary number:



Example 2: Conversion of decimal 45 to its equivalent binary number:

| | | | |
|---|---|---|---|
| | | | Top |
| $45 \div 2 = 22$ | with a remainder | 1 | (LSB) |
| $22 \div 2 = 11$ | with a remainder | 0 | |
| $11 \div 2 = 5$ | with a remainder | 1 | |
| $5 \div 2 = 2$ | with a remainder | 1 | |
| $2 \div 2 = 1$ | with a remainder | 0 | |
| $1 \div 2 = 0$ | with a remainder | 1 | (MSB) |
| | | | Bottom |

Reading the remainders from bottom to top, the binary number for the decimal number 45 is $101101_2$.

$\therefore \quad 45_{10} = 101101_2$ **Ans.**

### 5.3.2. Converting Fractional Numbers from Decimal-to-Binary

We have already seen in the last article that the repeated division-by-2 method can be used to convert integer (or whole number) decimal to its equivalent binary. Decimal fractions can be converted to binary by repeated multiplication-by-2. Following is the procedure that is used for conversion:

Step 1. Begin by multiplying the given decimal fraction by 2 and then multiplying each resulting fractional part of the product by 2.

Step 2. Repeat step 1 until the fractional product is zero or until the desired number of decimal places is reached.

Step 3. The carried bits or carries generated by the multiplication produce the binary number. The first carry produced is the most-significant bit (MSB) and the last carry is the least significant bit (LSB). In other words, reading from top to bottom constitutes the required fractional binary.

In order to illustrate the procedure, consider the conversion of 0.3125 to its equivalent binary Fraction

| | | | |
|---|---|---|---|
| | | | Top |
| $0.3125 \times 2 = 0.625$ | with a carry | 0 | MSB |
| $0.625 \times 2 = 1.25 = 0.25$ | with a carry | 1 | |
| $0.25 \times 2 = 0.50 = 0.50$ | with a carry | 0 | |
| $0.50 \times 2 = 1.00 = 0$ | with a carry | 1 | LSB |
| | | | Bottom |

Since the fractional part is zero, so we will stop the repeated multiplication. Reading from top to bottom, the required binary number is 0101.

$$0.3125_{10} = 0.101_2$$

## 5.4. BINARY TO DECIMAL CONVERSION

### 5.4.1 Converting an Integer (or Whole) Binary Number to Decimal Number

Following is the procedure for converting an integer (or whole) binary number to its equivalent decimal number.

Step 1. Write the binary number.

Step 2. Directly under the binary number, write the position values or weights of each bit working from right to left.

Step 3. If a zero appears in a digit position, cross-out the weight for that position.

Step 4. Add the remaining weights to obtain the decimal equivalent.

Given the binary number = $01010110_2$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Step 1. | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| Step 2. | $2^7$ | $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
| Step 3. | $\cancel{2^7}$ | $2^6$ | $\cancel{2^5}$ | $2^4$ | $\cancel{2^3}$ | $2^2$ | $2^1$ | $\cancel{2^0}$ |

Step 4. $2^6 + 2^4 + 2^2 + 2^1 = 64 + 16 + 4 + 2 = 86_{10}$

## 5.4.2. Fractional Binary-to-Decimal Conversion

We have already discussed the integer binary-to-decimal conversion. Let us now see how a binary fraction can be converted into corresponding decimal equivalents. Consider for example the conversion of fractional binary number 0.1010 to decimal.

| | | | | | |
|---|---|---|---|---|---|
| Step 1. | 0 . | 1 | 0 | 1 | 0 |
| | | ↓ | ↓ | ↓ | ↓ |
| Step 2. | | $2^{-1}$ | $2^{-2}$ | $2^{-3}$ | $2^{-4}$ |
| Step 3. | | $2^{-1}$ | $\cancel{2^{-2}}$ | $2^{-3}$ | $\cancel{2^{-4}}$ |

Step 4. $2^{-1} + 2^{-3} = 0.5 + 0.125 = 0.625_{10}$ **Ans.**

## 5.5. OCTAL-TO-DECIMAL CONVERSION

Following is the procedure to convert an octal number to its decimal equivalent.

Step 1. Write the octal number.

Step 2. Directly under the octal number write the position weight of each digit working from right to left.

Step 3. Multiply each octal digit by its position weight and take sum of the products.

In order to illustrate the method, consider an example, of converting 2374.

| | | | | |
|---|---|---|---|---|
| Step 1. | 2 | 3 | 7 | 4 |
| | ↓ | ↓ | ↓ | ↓ |
| Step 2. | $8^{+3}$ | $8^{+2}$ | $8^{+1}$ | $8^{0}$ |

Step 3.
$$= (2\times 8^3) + (3\times 8^2) + (7\times 8^1) + (4\times 8^0)$$
$$= (2\times 512) + (3\times 64) + (7\times 8) + (4\times 1)$$
$$= 1024 + 192 + 56 + 4$$
$$= 1276_{10}$$

$\therefore \quad 2374_8 = 1276_{10}$

## 5.6 HEXADECIMAL-TO-DECIMAL CONVERSION

Following is the procedure to convert a hexadecimal number to its equivalent decimal number:

Step 1. Write the hexadecimal number.

Step 2. Directly under the hexadecimal number, write the position weight of each digit working from right to left.

Step 3. Multiply the decimal value of each hexadecimal digit by its position weight and take sum of the products.

Hexa-decimal number: 2A6

| | | | |
|---|---|---|---|
| Step 1. | 2 | A | 6 |
| | ↓ | ↓ | ↓ |
| Step 2. | $16^{+2}$ | $16^{+1}$ | $16^{+0}$ |

Step 3.
$$(2\times 16^{+2}) + (A\times 16^{+1}) + (6\times 16^0)$$
$$= (2\times 256) + (10\times 16) + (6\times 1)$$
$$= 678$$

$\therefore \quad 2A6_{16} = 678_{10}$ **Ans.**

**I. Decimal number to Hexadecimal number.**

The decimal number id divided by 16 and the remainders are converted in to Hexa notations. The readings are taken from bottom to top.

| | | |
|---|---|---|
| 16 \| 2047 | | |
| 16 \| 127 — | 15 — F | ↑ |
| 7 — | 15 — F | |

$(2047)_{10} = (7FF)_{16}$

**II. Hexadecimal number or decimal number:**

Multiply each hexadecimal digit by its weight and add the result.

$$(3A45)_{16} = 3\times 16^3 + 10\times 16^2 + 4\times 16^1 + 5\times 16^0$$
$$= (14917)_{10}.$$

**III. Binary number into Hexadecimal number:**

Split the given number into 4-digit groups (supplying 0's if necessary)and then give the hex value.

Ex: $(1011010111011)_2$

→ 0010 1101 0111 1011

↓ ↓ ↓ ↓

2 *D* 7 *B*

**IV. Hexadecimal to binary number:**

Convert each hexadecimal digit into its 4-bit equivalent using to hex number.

$(7CF)_{16}$→ 7 C F

↓ ↓ ↓

0111 1100 1111

$= (011111001111)_2$

$(B2D)_{16}$→ *B* 2 D

↓ ↓ ↓

1011 0010 1101

$= (101100101101)_2$

## 5.7. BINARY OPERATIONS

### 5.7.1. BINARY ADDITION

Binary addition is much easier than the decimal addition. Following are the rules to perform binary addition.

Rule1: 0+0 =0 carry 0

Rule2: 1+0=1 carry 0

Rule3: 0+1=1 carry 0

Rule4: 1+1=0 carry 1

Example:

| Binary | Decimal | Binary. | Decimal | Binary | Decimal |
|---|---|---|---|---|---|
| 110 | (6) | 1011 | (11) | 11 | (3) |
| + 1011 | (9) | + 111 | (7) | + 101 | (5) |
| 1111 | (15) | 10010 | (18) | 1000 | (8) |

### 5.7.2. BINARY SUBTRACTION:

Following are the rules to perform binary subtraction.

Rule1: 0-0=0 No borrowing

Rule2: 1-0=1 No borrowing

Rule3: 0-1=0 No borrowing

Rule4: 1-1=1 with a borrow of 1

Example:

| | | |
|---|---|---|
| 1101 | 1010 | 11011 |
| -1010 | -1001 | -10111 |
| 0011 | 0001 | 00100 |

## 5.7.3. BINARY MULTIPLICATION AND DIVISION

The complete binary multiplication table is as follows

Rule1: 0 x 0 = 0

Rule2: 0 x 1 = 0

Rule3: 1 x 0 = 0

Rule4: 1 x 1 = 1

The rules of binary division are given below:

Rule 1: 0/1 = 0

Rule 2: 1/1 = 1

## 5.8. 1'S and 2'S COMPLEMENTs

Complement of a Number: In digital operation, there are two types of complements of a binary number which are used for complemental subtraction.

### (i) 1's Complement:

This is also known as radix-minus-one component. The 1's component of a binary number is obtained by changing its each zero (0) into one (1) and each one (1) into zero (0).

For example, 1's complement of $(1101)_2 = (0010)_2$

### (ii) 2's Complement:

This is also known as true complement. The 2's complement of a binary number is obtained by adding one (1) to its l's complement. Thus,

2's complement = 1 +'s complement

To find out the 2's complement of $(1011)_2$,

1's complement of $(1011)_2 = (0100)_2$

add 1 to this result : $(0100)_2 +1 = (0101)_2$

Now, we get $(0101)_2$.

Hence, 2's complement of $(1011)_2= (0101)_2$

## 5.8.1. 1'S COMPLEMENTAL SUBTRACTION

**For 1's complemental subtraction,**

(i) First find the 1's complement of the subtrahend number.

(ii) Now we add this 1's complement to the minuend.

(iii) If there is a 1 carry in the most significant position, then this carry is removed and perform the end-around carry of the last 1 (add carry 1 to the previous result).

(iv) If there is no end-around carry (i.e., 0 carry), then the answer is re-complemented (1's complement) and put a negative sign for the final result.

## 5.8.2. 2'S COMPLEMENTAL SUBTRACTION

**For 2's complemental subtraction,**

i) Change all 0's to 1's and 1's to 0's of subtrahend to get the 1's complement

ii) then add 1 to get 2's complement.

iii) add the 2's complemental to minuend.

iv) If carry is one (1), then drop the carry. The answer is positive and needs no re-complementing.

v) If there is no carry, we re-complement (2's complement) the result and attach a negative sign to it.

## 5.9. BOOLEAN ALGEBRA

In our daily life, we have so many two valued problems like: is it true or false? good or bad? right or wrong? etc. First of all, Aristotle tried to find out precise methods for getting the truth for such two valued or binary problems based on a set of assumptions. Further, De Morgan proposed that the logic has connection with mathematics. But George Boole (1815-1864) tried to perform a mathematical analysis of logic and constructed a logical algebra. This algebra is commonly known as Boolean algebra. Thus, Boolean algebra is a system of mathematical logic. It differs from both ordinary algebra and the binary number system.

In Boolean Algebra, only two logic variables are permitted, i.e., true or false, usually written as 1and 0. As digital system works in binary, i.e., involves only two possible states, the Boolean Algebra has been used extensively in digital electronics. So, Boolean Algebra is ideal for the design and analysis of logic circuits used in computers.

The three logical operators to form a logical function are:

1. OR operator, 2. AND operator and 3. NOT operator

### 1. OR Operator

In Boolean algebra, the OR operator is indicated with a plus (+) sign. Similar to addition. Therefore, A + B = C states that if A is true (1) OR B is true (l), then C will be true (1). C will be false (0) when both A and B are false (0). The different combinations of A and B are shown below

| A | + | B | = | C |
|---|---|---|---|---|
| 0 | + | 0 | = | 0 |
| 1 | + | 0 | = | 1 |
| 0 | + | 1 | = | 1 |
| 1 | + | 1 | = | 1 |

### 2. AND Operator

In Boolean algebra the AND operator is defined by the use of multiplication (x). For example, A x B = C means that if A is true AND B is true, then C will be true. Except this, C will be false. The different combinations of A and B are

| A | × | B | = | C |
|---|---|---|---|---|
| 0 | × | 0 | = | 0 |
| 1 | × | 0 | = | 0 |
| 0 | × | 1 | = | 0 |
| 1 | × | 1 | = | 1 |

### 3. NOT Operator

In Boolean algebra, the NOT operator is defined by the use of a bar (-). Thus the NOT operator represents the inversion. Therefore, if $\bar{A}$ is true (1), then A will be false (0) and vice-versa.

### 5.9.1 BOOLEAN LAWS

#### (1) Laws of Complementation (NOT Laws)

The term complement means to invert. So, in Boolean Algebra, change 1 to 0 or 0 to 1. Thus, we have

Rule 1: $\bar{0} = 1$ Rule 3: If A = 0, then $\bar{A} = 1$

Rule 2: $\bar{1} = 0$ Rule 4: If A = 1, then $\bar{A} = 0$

So, for a logic variable X, we have $X \cdot \bar{X} = 0$ and $X + \bar{X} = 1$

#### (2) OR Laws

The OR operation is represented by + sign. If A and B are inputs and C is output, then

$$A + B = C$$

We know that $0 + 0 = 0$; $0 + 1 = 1$; $1 + 0 = 1$; $1 + 1 = 1$.

From these expressions, if both the inputs are 0, then output will be zero and if any input or both inputs is 1, then output will be 1.

Law 1. $A + 0 = A$, Law 3. $A + A = A$

Law 2. $A + 1 = 1$ Law 4. $A + \bar{A} = 1$

#### (3) AND Laws

The AND operation is represented by multiplication. $A \cdot B = C$ shows the AND laws. So

$0 \cdot 0 = 0$, $0 \cdot 1 = 0$

$1 \cdot 0 = 0$, $1 \cdot 1 = 1$

So, if any input is zero or both inputs are zero, then output will be zero while if both the inputs are 1, then only the output will be one.

Law 1. $A \cdot 0 = 0$, Law 3. $A \cdot A = A$

Law 2. $A \cdot 1 = A$ Law 4. $A \cdot \bar{A} = 0$

#### (4) Commutative Laws

There are two commutative law. These laws allow change in the position of variables in OR and AND expressions. These are

$$A + B = B + A$$
$$A \cdot B = B \cdot A$$

#### (5) Associative Laws

There are two associative laws. These laws allow removal of bracket from logical expression and regrouping of variables. These laws can be stated as

Law 1 : $A + (B + C) = (A + B) + C$

Law 2: $A \cdot (B \cdot C) = (A \cdot B) \cdot C$

#### (6) Distributive Laws

There are three distributive laws. These laws show that we can expand expressions by multiplying terms as in ordinary algebra. The distributive laws are:

Law 1: $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$

Law 2: $A + (B \cdot C) = (A + B) \cdot (A + C)$

Law 3: $A + (\bar{A} \cdot B) = A + B$

**(7) Absorptive Laws**

There are three absorptive laws. These are

Law 1: $A + (A \cdot B) = A$

Law 2: $A \cdot (A + B) = A$

Law 3: $A \cdot (\bar{A} + B) = A \cdot B$

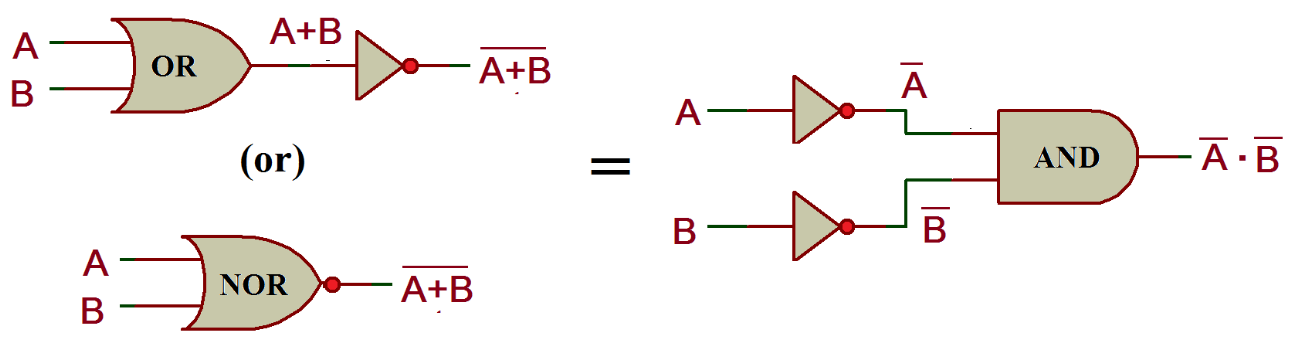
ALL Boolean laws are

| | |
|---|---|
| 1. $A + 0 = A$ | 11. $\bar{\bar{A}} = A$ |
| 2. $A + 1 = 1$ | 12. $A + B = B + A$ |
| 3. $A + A = A$ | 13. $A \cdot B = B \cdot A$ |
| 4. $A + \bar{A} = 1$ | 14. $A + A \cdot B = A$ |
| 5. $A \cdot 0 = 0$ | 15. $A + \bar{A}B = A + B$ |
| 6. $A \cdot 1 = 1$ | 16. $(A + B) + C = A + (B + C)$ |
| 7. $A \cdot A = A$ | 17. $(A \cdot B) \cdot C = A \cdot (B \cdot C)$ |
| 8. $A \cdot \bar{A} = 0$ | 18. $A \cdot (B + C) = A \cdot B + A \cdot C$ |
| 9. $\bar{0} = 1$ | 19. $A + B \cdot C = (A + B) \cdot (A + C)$ |
| 10. $\bar{1} = 0$ | 20. $A \cdot (\bar{A} + B) = A\,B$ |

**5.10. DEMORGANS LAWS:**

**1st law:** The complement of sum of two (or) more variables is equal to the product of complements of the variables.

i.e, $\overline{A+B} = \bar{A}\,.\,\bar{B}$




**Proof:** the only two values for the variables A and B are 0 and 1

**Case i:** if A= 0, B = 0.

L.H.S = $\overline{0+0}=1$

R.H.S = $\bar{0}.\bar{0}=1.1=1$

L.H.S = R.H.S

**Case ii:** if A=1, B = 0.

L.H.S = $\overline{1+0} = \overline{1} = 0$

R.H.S = $\overline{1}.\overline{0} = 0.1 = 0$

L.H.S = R.H.S

**Case iii:** if A = 0, B = 1.

L.H.S = $\overline{0+1} = \overline{1} = 0$

R.H.S = $\overline{0}.\overline{1} = 1.0 = 0$

L.H.S = R.H.S

**Case iv:** if A=1, B=1.

L.H.S = $\overline{1+1} = \overline{1} = 0$

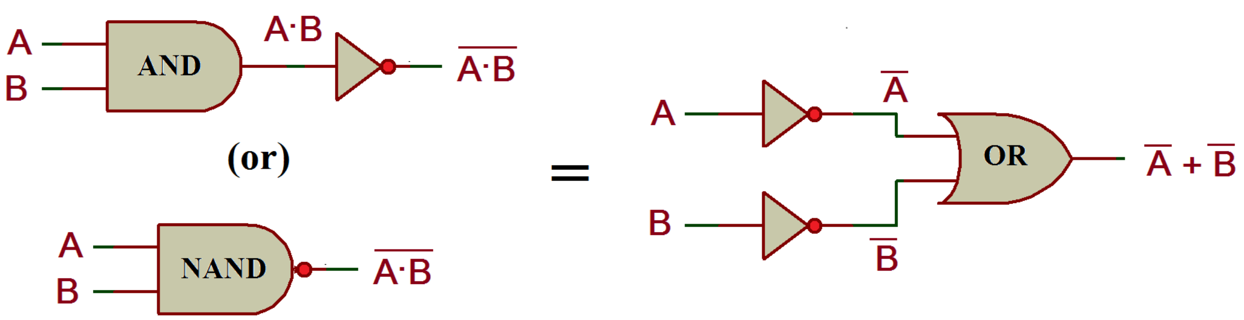
R.H.S = $\overline{1}.\overline{1} = 0.0 = 0$

L.H.S = R.H.S

Thus the first law is verified

__2$^{nd}$law:__ The complement of the product of two (or) more variables is equal to sum of complements of the variables.

i.e, $\overline{A.B} = \overline{A} + \overline{B}$.




**Proof:** the only two values for the variables A and B are 0 and 1

**Case i:** if A = 0, B = 0.

L.H.S = $\overline{0.0} = \overline{0} = 1$

R.H.S = $\overline{0} + \overline{0} = 1+1 = 1$

L.H.S = R.H.S

**Case ii:** if A=1, B = 0.

L.H.S = $\overline{1.0} = \overline{0} = 1$

R.H.S = $\overline{1}+\overline{0} = 0 +1 = 1$

L.H.S = R.H.S

**Case iii:** if A= 0, B =1.

L.H.S = $\overline{0.1} = \overline{0} = 1$

R.H.S = $\overline{0}+\overline{1} = 1+0 = 1$

L.H.S = R.H.S

**Case iv:** if A = 1, B = 1.

L.H.S $=\overline{1.1}=\overline{1}=0$

R.H.S $=\overline{1}+\overline{1}=0+0=0$

L.H.S = R.H.S

Thus the second law is verified

## <u>5.11. BASIC LOGIC GATES</u>

Circuits which are used to process digital signals are called logic gates. Gate is a digital circuit with one or more input voltages but only one output voltage. By connecting these gates in different ways, we can build circuits that can perform arithmetic and other functions that are associated with the human brain. The gates are often called logic circuits. The most basic gates are called the AND gate, the OR gate and the NOT gate.

Logic gates are of two types- combinational and sequential. In combinational gates, the output at any instant depends upon the inputs at that instant. Here the previous input does not have any effect on the output. Eg: OR, NOT, NAND, NOR, XOR. But sequential gates, the output depends upon the order or sequence in which the inputs are applied. Hence, sequential gates possess a memory function. Eg: Flip-Flops, counters and Registers.

For analysis and design of digital circuits, 3 basic gates are used for fundamental operations.

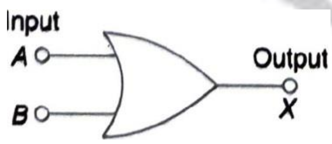
(1) OR gate for addition (+)

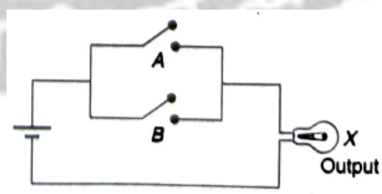
(2) AND gate for multiplication (x) and

(3) NOT gate for inverting (represented by a bar over the variable).

### <u>5.11.1. OR gate</u>

An OR gate has two or more input signals but only one output signal. In OR gate, output voltage is high if any or all the input voltages are high (or) Output is low only when the two inputs are low. The symbolic representation of OR gate is shown in fig.



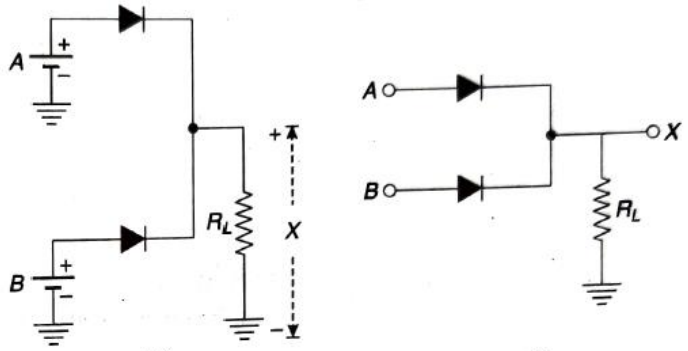

The electrical equivalence of the OR logic can be explained with circuit diagram.

In the electrical circuit, the switches A and B are connected in parallel with each other. The lamp (output) will glow only when A or B or both switches are closed.

The logic is indicated by expression A + B = X

| Input | | Output |
|---|---|---|
| A | B | $X = A + B$ |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

(a)




(b) (c)

The construction of OR gate can be done with two diodes and the operation of the OR gate as follows.

(i) When both A and B are at zero levels or grounded (low, 0), the two diodes are non-conducting (reverse biased). No current flow to the output. So the output voltage is low (0).

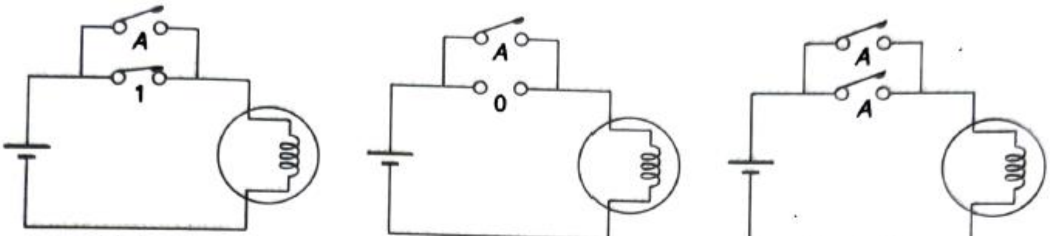
(ii) When A is 1ow (0) and B is high (1), diode B is forward biased so that Current flows through $R_L$, and output, X is high (1).

(iii) When A is high (1) and B is 1ow (0), diode A is forward biased so that Current flows through $R_L$, and output, X is high (1).

(iv) when both A and B are high (1), both the diodes are conducting (Forward biased) and hence X is high (1).

Based on the above logic, we have three Boolean algebraic laws.

i.e., A + 1 =1, A + 0 = A and A + A = A

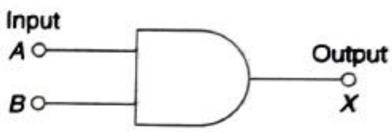



(a) $A + 1 = 1$ (b) $A + 0 = A$ (c) $A + A = A$

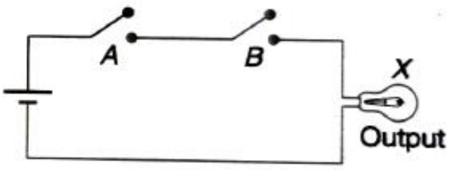
These laws can be verified in the cases when A = 0 and A = 1.

**5.11.2. AND gate**

An AND gate has also two or more input signals but only one output signal. In AND gate, output voltage is low if any or all the input voltages are low (or) Output is high only when all the inputs are high. The operation of an AND gate is similar to multiplication (.). The symbolic representation of AND gate and its electrical equivalent diagram is as shown in fig.




(a)




(b)

In the electrical equivalent circuit, the switches A and B are connected in series with each other.

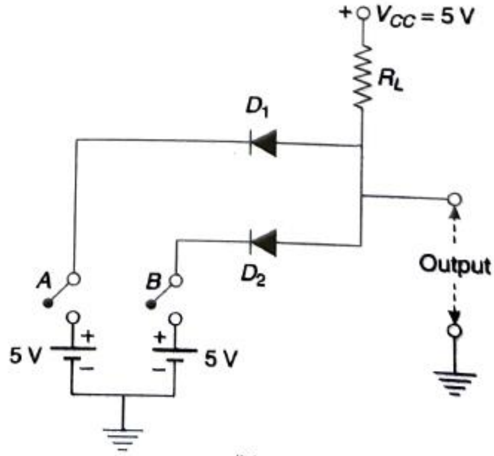
The lamp (output) will glow only when A and B both switches are closed.

The logic is indicated by expression $A \cdot B = X$

The following is the construction of AND gate using two diodes and the corresponding Truth table is also provided.

| Input | | Output |
|---|---|---|
| $A$ | $B$ | $X = A \bullet B$ |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

(a)




(b)

**Operation of AND gate:**

(i) When both A and B are zeros i.e., the cathode of each diode is grounded, the two diodes are conducting as they are forward biased. Now the diodes have very less resistance, So the output voltage (current x resistance) w.r.t. earth is zero. The whole voltage $V_{CC}$ will be dropped across $R_L$ only. So the output voltage is low (0).

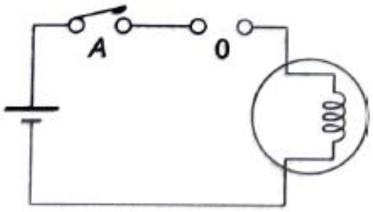
(ii) When A is zero (low) and B = 5V (high), the diode $D_1$ is grounded and diode $D_2$ is reversed-biased. Due to reverse-biasing, the diode $D_2$ stops conducting. The diode $D_1$ will now conduct due to forward-biasing. Therefore, the output will be zero as above.

(iii) When A is 5 V (high) and B is zero (low). In this case, $D_1$ in forward biased (conducting) and $D_2$ in reverse biased (non-conducting). Here also due to $D_1$ less resistance appears at output terminal. Hence, output is zero.

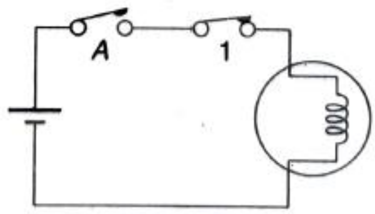
(iv) When A and B are high, both the diodes are non-conducting. Since the diodes are off, there is no current through $R_L$. Now the supply voltage $V_{CC}$ = 5 V appear at the output. Therefore, output is high.

Based on the above logic, we have three Boolean algebraic laws.

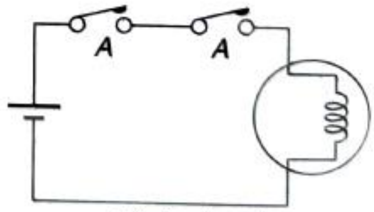
i.e., $A \cdot 0 = 0$, $A \cdot 1 = A$ and $A \cdot A = A$




(a) $A \cdot 0 = 0$




(b) $A \cdot 1 = A$

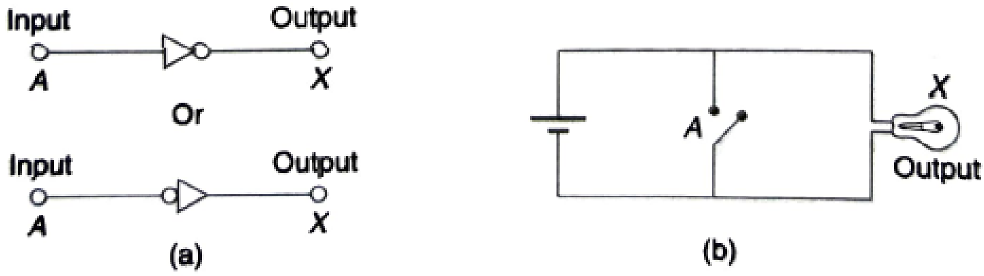



(c) $A \cdot A = A$

These laws can be verified in the cases when A = 0 and A = 1.

### 5.11.3. NOT gate

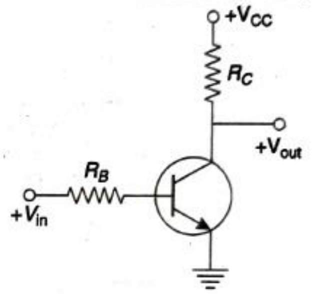
The NOT gate is a gate with only one input and one output. Here, always output is the complement of the input. So the NOT gate is also called an inverter. When the input voltage is high, the output is low and vice-versa. The following is the symbolic representation of NOT gate and its electric equivalent circuit.




From the electrical analog of NOT gate, when switch A is closed (input is high) the bulb will not glow (i.e., output is low) and vice-versa.

NOT Circuit Operation

| Input | Output |
|---|---|
| $A$ | $X = \overline{A}$ |
| Low 0<br>High 1 | High 1<br>Low 0 |



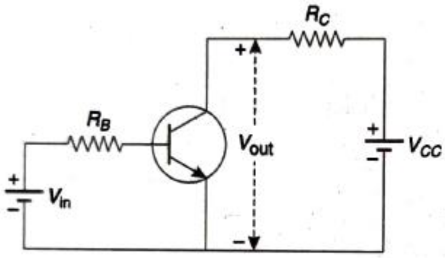
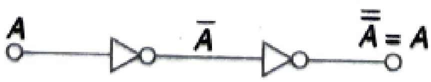




Fig. shows the common-emitter circuit which is used as a NOT gate. The circuit operation is quite simple. When $V_{in}$ is high, the transistor is driven into saturation so that $V_{out}$ is low. If $V_{in}$ is low, i.e., base voltage is not present, the transistor is in cut off so that $V_{out}$ is high. Hence, we see that whenever input is high, the output is low and vice-versa which verifies the NOT Truth-table.

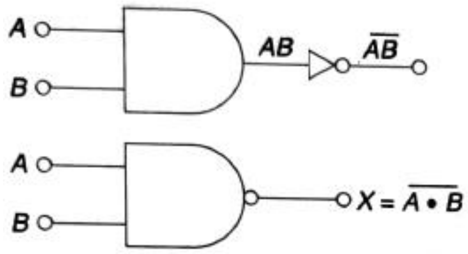
The NOT operation is represented by a line drawn over the top of the input. The negation of A is $\bar{A}$, called NOT A or sometimes A-bar.

The NOT laws are quite simple $\overline{0} = 1$ and $\overline{1}$ = 0 Also, $\bar{\bar{A}}$ =  A

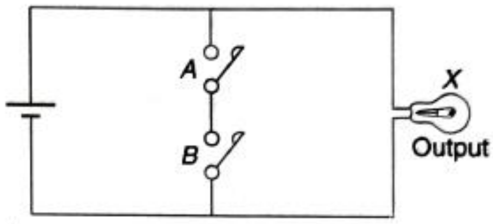

### 5.11.4. NAND gate

NAND gate is A logic circuit in which a NOT gate followed by AND gate. So, NAND gate is a combination of AND gate and a NOT gate. The symbol of a NAND gate and corresponding equivalent circuit diagram is as shown in fig. Here the bubble on the output reminds us of the inversion after ANDing. Mathematically it is expressed as

$$X = \overline{A \cdot B}$$



(a)



(b)

From the circuit, the bulb will glow only when either A and B or both are opened. If both are closed then short circuited, so bulb will not glow.

(i) When A = 0, B = 0 then AB = 0 and $X = \overline{AB} = 1$

(ii) When A = 0, B = 1 then AB = 0 and $X = \overline{AB} = 1$

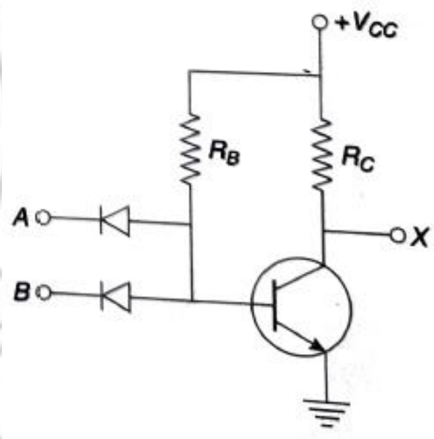
(iii) When A = 1, B = 0 then AB = 0 and $X = \overline{AB} = 1$

(iv) When A = 1, B = 1 then AB = 1 and $X = \overline{AB} = 0$

| Input | | Output |
|---|---|---|
| A | B | $X = \overline{A . B}$ |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

**5.11.4.1. Circuit of NAND gate**

The output of a NAND gate is given by X = A .B. Fig. shows circuit of NAND gate. Here a diode AND circuit is connected to a transistor NOT circuit which gives the NAND circuit. It can be seen that when the two inputs are zero, the transistor will not conduct. So the output will be $+V_{CC}$. On the other hand, when positive voltages are applied to the two inputs the transistor will conduct into saturation region. Now the output will drop to zero. In this way the circuit behaves as a reverse AND circuit and hence called as NAND circuit. As we already know OR and AND gates can be built by using diodes or transistors but a NOT gate can be built by using transistors only.

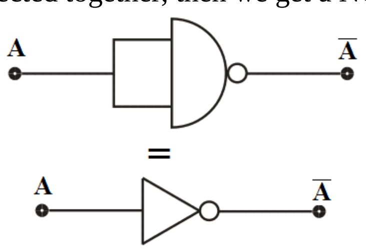



**5.11.4.2. NAND GATE AS A UNIVERSAL GATE**

The NAND gate is called a universal gate since any logic circuit (OR, AND, NOT) can be built by using NAND gate.

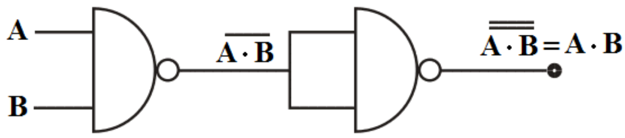
**1 As NOT gate**

If the two inputs of NAND gate are connected together, then we get a NOT gate.



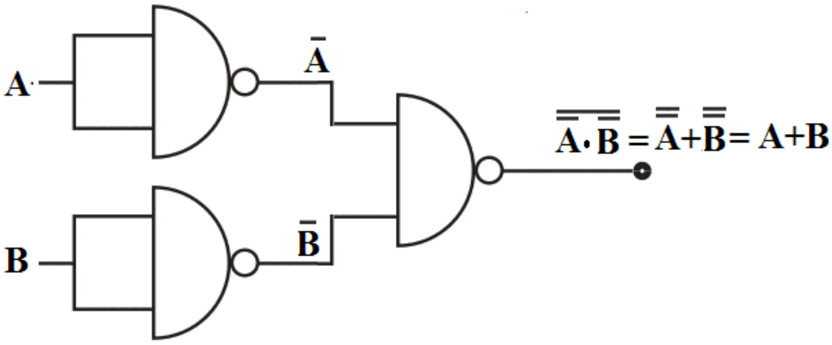

**2. As AND gate**

The AND gate can be produced by connecting two NAND gates in series.

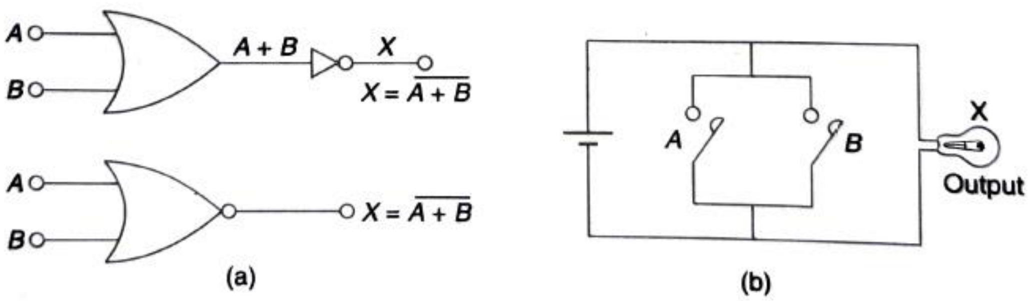



**3. As OR gate**

OR gate can be produced by three NAND gates as shown in fig. It is important to mention here that OR function may not be clear from this figure because De Morgan's theorem is needed to prove that $\overline{\bar{A} \cdot \bar{B}} = A + B$




## 5.11.5. NOR GATE




NOR gate coming from the words NOT and OR. NOR gate = OR gate + NOT gate. From the figure, the bubble in the output signifies that inversion takes place after the OR ring.

As is evident from this figure the lamp will not glow (output is zero) when either of the inputs A or B is high.

Following is the truth table for NOR gate

| Input | | Output |
|---|---|---|
| A | B | X |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

The output of a NOR gate can be written as $X = \overline{A + B}$.

(i) When A = 0, B = 0, then A + B = 0 $\Rightarrow X = \overline{A + B} = 1$

(ii) When A = 0, B = 1, then A + B = 1 $\Rightarrow X = \overline{A + B} = 0$

(iii) When A = 1, B = 0, then A + B = 1 $\Rightarrow X = \overline{A + B} = 0$

(iv) When A = 1, B = 1, then A + B = 1 $\Rightarrow X = \overline{A + B} = 0$

### 5.11.5.1. Circuit of NOR gate

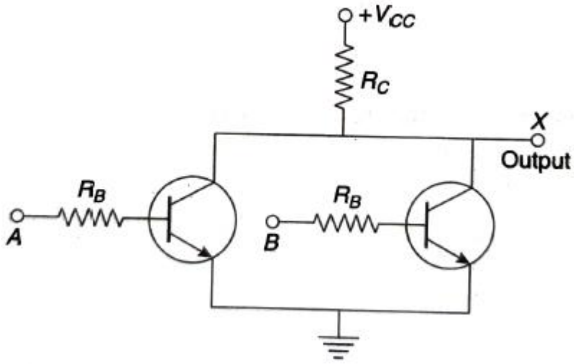


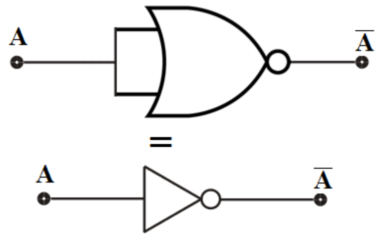

Fig. shows the circuit of a NOR gate. As seen from the figure, the output X is 1 only when both transistors are cut off, i.e., when A = 0 and B = 0. For any other condition of input one or both transistors saturate and the result is that the output X is 0.

### 5.11.5.2. NOR GATE AS A UNIVERSAL GATE

NOR gate is a universal gate because it can be used to perform the basic logic functions OR, AND and NOT.

**1. As NOT gate**

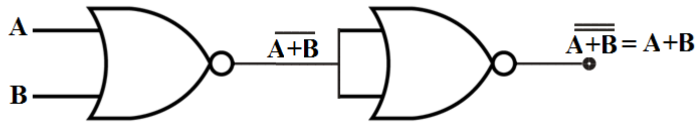
When the inputs of NOR gate are tied together, the output is A + A as shown in fig. By De Morgan's theorem this is equivalent to A (i.e., A + A = A). This is the function of NOT gate.




**2. As OR gate**

This gate can be realized by connecting output of a NOR gate to NOT gate. This is shown in fig.

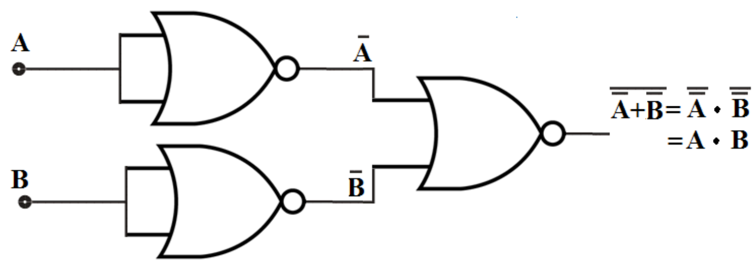
The output of NOR gate is A + B. This is inverted by NOT gate to give X = A+B. This is inverted by NOT gate to give X =A +B. This is a function of OR gate.




**3. As AND gate**

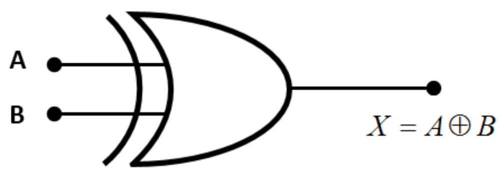
AND gate can be made out of three NOR gates. Here, two NOR gates are used to invert the inputs and the third one is used to combine the inverted inputs.

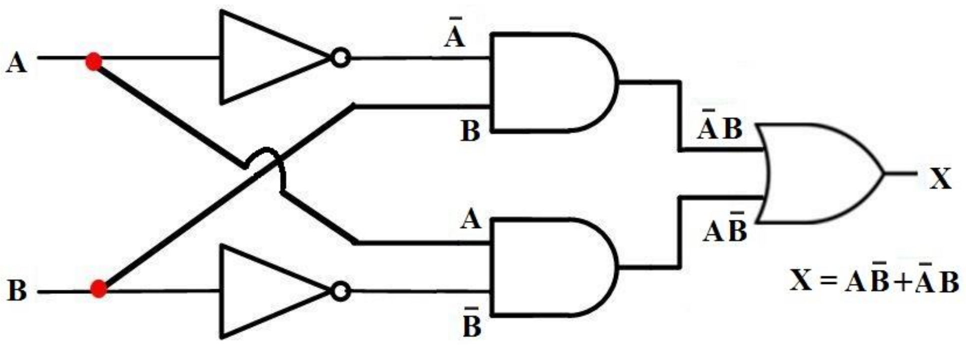
The output $A \cdot B$ is a function of AND gate. This can be proved with the help of De Morgan's law.




### 5.11.6. Exclusive OR Gate (XOR Gate):

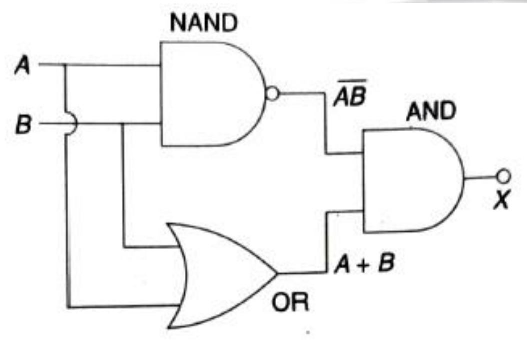
The symbolic representation of XOR Gate is shown in figure. There are two inputs and one output. The exclusive OR operation is denoted by $\oplus$. Hence the output is given by $X = A \oplus B$






$$X = A \oplus B = A.\overline{B} + \overline{A}.B$$

The other way to implement XOR logic are shown in fig




$$X = \overline{AB} \cdot (A + B)$$

The truth table for EX-OR Gate is shown below

**Truth Table:**

| A | B | $A.\overline{B}$ | $\overline{A}.B$ | $X = A \oplus B = A.\overline{B} + \overline{A}.B$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 |

Case 1: if A = 0, B=0, then output X = 0.

Case 2: if A = 0, B=1, then output X = 1.

Case 3: if A = 1, B=0, then output X = 1.

Case 4: if A = 1, B=1, then output X = 0

From the truth-table, output is 1 only when the inputs are different. XOR operation also called mod-2 addition.

**Applications of XOR gate:**

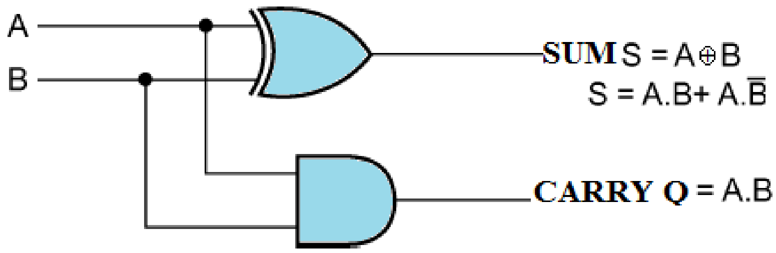
(1) XOR gate can be used as Binary to Gray code converter.

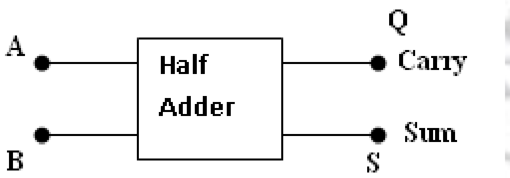
(2) It can be used as a parity checker.

(3) It is also used in Half-adder circuit.

## 5.12. HALF ADDER:

Half adder is an electronic logic circuit that adds two single bits and produce a sum and a carry to be used in the next higher position. It consists of two inputs A and B and two outputs carry **Q** and sum **S**. It does not accept a carry input **or** previous carry **Q'**. To accept a previous carry (Q') a full adder is used. Here sum represents the output of Exclusive OR Gate and carry represents the output of AND Gate. The standard symbol for half adder is shown in figure.




**Truth table:**

| A | B | Carry Q | Sum S |
|---|---|---|---|
| **0** | **0** | **0** | **0** |
| **1** | **0** | **0** | **1** |
| **0** | **1** | **0** | **1** |
| **1** | **1** | **1** | **0** |

**Case 1:** if A=0, B=0, then Q=0, S=0.

**Case 2:** if A=0, B=1, then Q=0, S=1.

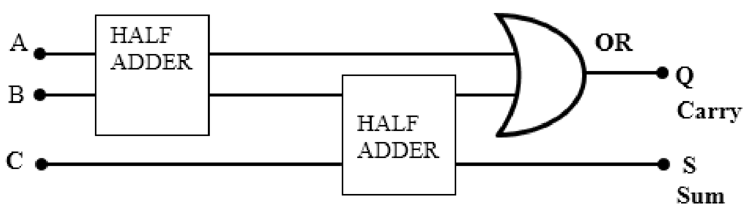
**Case 3:** if A=1, B=0, then Q=0, S=1.

**Case 4:** if A=1, B=1, then Q=1, S=0.

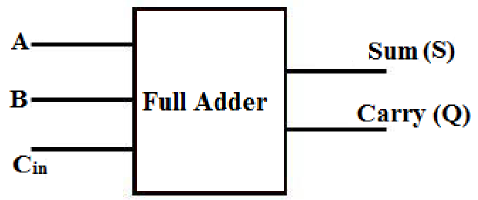
The circuit is called half-adder because it cannot accept a carry in from previous additions. For this purpose, we need a 3-input adder called full-adder.

## 5.13. FULL ADDER:

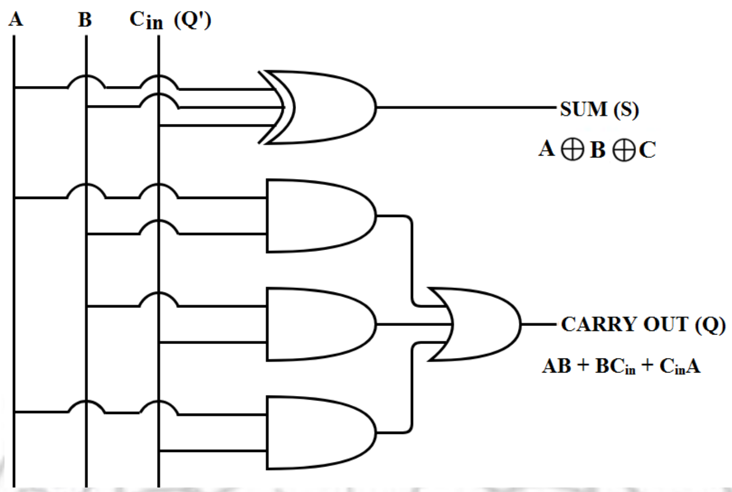
Half-adder can add only two lower significant bits. It cannot be used for addition of higher position bits because those have previous carry values also. We can define full-adder as a logic circuit that adds three bits and produce SUM (S) and CARRY OUT (Q). This CARRY OUT is taken as CARRY IN **(Q')** in the next addition. In this way we have 3 inputs for addition. Two are bits to be added and the third is CARRY IN **(Q')** generated from previous addition. In full adder two half adders and one OR-Gate over cascaded as shown in figure.




The standard symbol for FULL ADDER is as follows.




Implementation of full adder using logic gates as follows



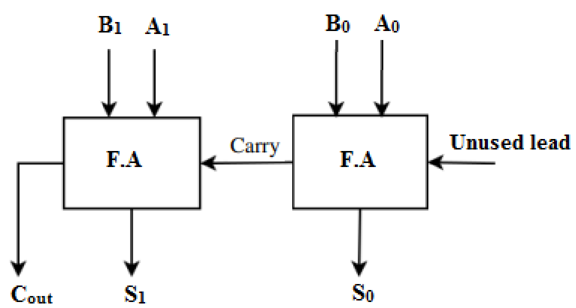

The truth table for FULL ADDER is given below

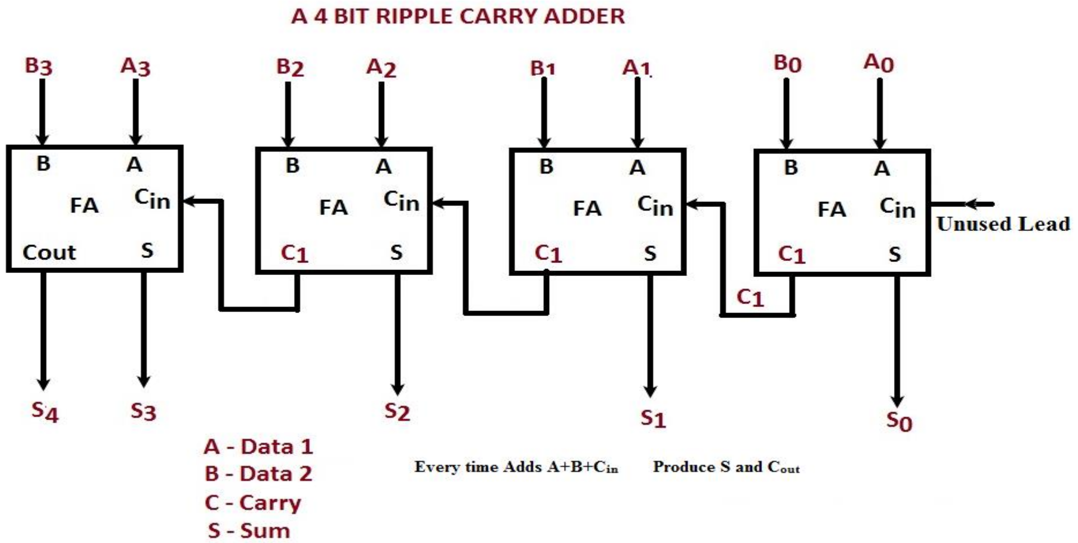
| Inputs | | | Outputs | |
|---|---|---|---|---|
| A | B | $C_{in}$ (Q') | Sum (S) | Carry (Q) |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

## 5.14. PARALLEL BINARY ADDER

A single full adder is capable of adding two one-bit numbers and an input CARRY. For addition of binary numbers with more than one bit, additional full-adders are required. Thus, for n-bit numbers, n füll-adders are required. For implementing the addition of higher order binary numbers higher-order adder are needed. Consider a two –bit adder for two-bit binary number. The CARRY output of each adder is connected to the CARRY input of the next higher-order adder as shown in fig.

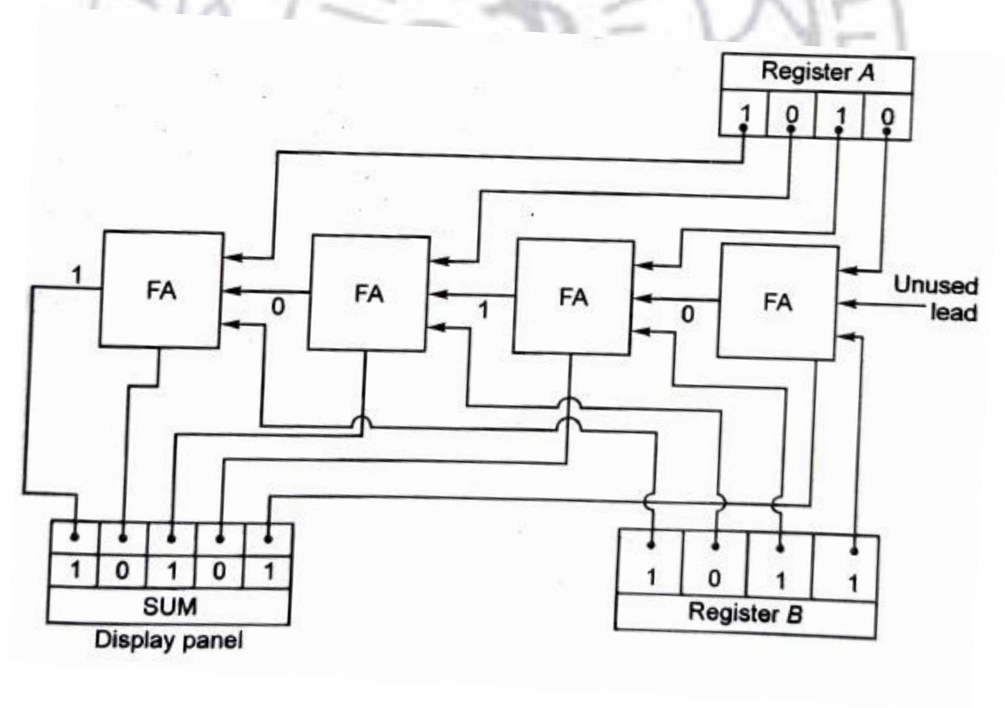
**Four-bit parallel adder**






A four-bit binary parallel adder circuit is shown in fig. The two numbers being added are $A_3$, $A_2$, $A_1$, $A_0$ and $B_3$, $B_2$, $B_1$, $B_0$. Their sum is $S_4$, $S_3$, $S_2$, $S_1$, and $S_0$, as shown in fig.

The full-adder circuit in each position has three inputs: an A bit, a B bit and a $C_{in}$ (CARRY IN) bit and it produces two outputs: a SUM bit and a CARRY OUT bit. For example, full-adder $FA_0$ has inputs $A_0$, $B_0$ and $C_{in}$. It produces output $S_0$ and $C_1$. Here carry $C_1$ is forwarded for next full-adder. The procedure is repeated for other full adders.




**Operation of 4-Bit Adder**

(i) The first adder performs 0 + 1 binary addition. This is gives a sum of l and a carry of 0.

(ii) 1 and 1 are supplied from two registers A and B simultaneously. The sum 1 appears on the display panel and carry 0 is passed to the next full-adder.

(iii) The second adder: adds 1 + 1 + 0 (carry), produce sum 0 with carry 1.

(iv) The third adder performs 0 + 0 + l (carry) and produce sum 1 with carry 0.

(v) The fourth adder adds 1 + 1 + 0 (carry) and produce sum 0 with carry 1 (both of which appear on display unit).

(vi) Therefore, the final number appears as 10101

# EXCERCISES ON NUMBER CONVERSION

**Conversion of Binary into decimal System:**

1. $(1011)_2$ =
2. $(1111)_2$ =
3. $(1101001)_2$ =
4. $(1011101)_2$ =
5. $(11010011)_2$ =
6. $(0.101)_2$ =
7. $(0.1001)_2$ =
8. $(1011.101)_2$ =
9. $(10010.1011)_2$=

**Conversion of Decimal to Binary system:**

1. $(11)_{10}$ =
2. $(57)_{10}$ =
3. $(45)_{10}$ =
4. $(211)_{10}$=
5. $(0.625)_{10}$=
6. $(0.5625)_{10}$=
7. $(57.59375)_{10}$=
8. $(47.5625)_{10}$ =